

Integrales indefinidas

ACTIVIDADES

1. Página 266

$$f(x) = 5x^4 - 3x^2 + 2 \rightarrow F(x) = x^5 - x^3 + 2x + 3$$

$$f(x) = x^4 + x^3 - 2 \rightarrow F(x) = \frac{x^5}{5} + \frac{x^4}{4} - 2x + 1$$

$$f(x) = 12x^2 + 6x \rightarrow F(x) = 4x^3 + 3x^2$$

2. Página 266

a) $2x^4 - 2x^2 + 5x$

c) $3e^x + x$

b) $\frac{2}{5}x^4$

d) $3\sqrt{x}$

3. Página 267

a) $\int (3x^2 - 2x + 1)dx = x^3 - x^2 + x + k$

e) $\int (3 - e^{-x})dx = 3x + e^{-x} + k$

b) $\int (2\cos x)dx = 2\sin x + k$

f) $\int (5 - 4x - \cos x)dx = 5x - 2x^2 - \sin x + k$

c) $\int (4x^3 - \sin x)dx = x^4 + \cos x + k$

g) $\int (5e^x + 2\cos x)dx = 5e^x + 2\sin x + k$

d) $\int (2x - e^x)dx = x^2 - e^x + k$

h) $\int (7\sin x + 4\cos x)dx = -7\cos x + 4\sin x + k$

4. Página 267

a) $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx = F(x) + G(x) + k$

b) $\int [2f(x) - g(x)]dx = 2\int f(x)dx - \int g(x)dx = 2F(x) - G(x) + k$

c) $\int \left[\frac{1}{2}f(x) - 2g(x) \right]dx = \frac{1}{2}\int f(x)dx - 2\int g(x)dx = \frac{1}{2}F(x) - 2G(x) + k$

d) $\int [-f(x) + b \cdot g(x)]dx = -\int f(x)dx + b \cdot \int g(x)dx = -F(x) + b \cdot G(x) + k$

5. Página 268

a) $\int (x^2 + 3x - 2)dx = \frac{1}{3}x^3 + \frac{3}{2}x^2 - 2x + k$

b) $\int \frac{3}{x^3}dx = \int 3x^{-3}dx = -\frac{3}{2x^2} + k$

c) $\int 3\sqrt[4]{x^3}dx = \int 3x^{\frac{3}{4}}dx = \frac{12}{7}\sqrt[4]{x^7} + k = \frac{12}{7}x\sqrt[4]{x^3} + k$

6. Página 268

$$a) \int (1-x)^2 dx = -\int (-1)(1-x)^2 dx = -\frac{(1-x)^3}{3} + k$$

$$b) \int (1-x^2)^2 dx = \int (1-2x^2+x^4) dx = x - \frac{2}{3}x^3 + \frac{x^5}{5} + k$$

$$c) \int 2x\sqrt{x^2+3} dx \rightarrow \text{Tomamos } f(x) = x^2+3 \rightarrow f'(x) = 2x \rightarrow \int 2x\sqrt{x^2+3} dx = \frac{2}{3}\sqrt{(x^2+3)^3} + k$$

7. Página 269

$$a) \int x(x^2+3) dx = \frac{1}{2} \int 2x(x^2+3) dx = \frac{1}{2} \cdot \frac{(x^2+3)^2}{2} + k = \frac{(x^2+3)^2}{4} + k$$

$$b) \int (x-2)(x^2-4x+1)^3 dx = \frac{1}{2} \int (2x-4)(x^2-4x+1)^3 dx = \frac{1}{2} \cdot \frac{(x^2-4x+1)^4}{4} + k = \frac{(x^2-4x+1)^4}{8} + k$$

8. Página 269

$$a) \int \sqrt{2x-1} dx = \frac{1}{2} \int 2\sqrt{2x-1} dx = \frac{1}{2} \cdot \frac{2}{3} \sqrt{(2x-1)^3} + k = \frac{\sqrt{(2x-1)^3}}{3} + k$$

$$b) \int x\sqrt{x^2-2} dx = \frac{1}{2} \int 2x\sqrt{x^2-2} dx = \frac{1}{2} \cdot \frac{2}{3} \sqrt{(x^2-2)^3} + k = \frac{\sqrt{(x^2-2)^3}}{3} + k$$

$$c) \int e^{2x} dx = \frac{1}{2} \int 2e^{2x} dx = \frac{1}{2} e^{2x} + k$$

$$d) \int (x+1)e^{x^2+2x-1} dx = \frac{1}{2} \int (2x+2)e^{x^2+2x-1} dx = \frac{1}{2} e^{x^2+2x-1} + k$$

9. Página 270

$$a) \int \frac{4x}{2x^2+1} dx = \ln|2x^2+1| + k$$

$$b) \int \frac{x^2}{x^3+3} dx = \frac{1}{3} \int \frac{3x^2}{x^3+3} dx = \frac{1}{3} \ln|x^3+3| + k$$

$$c) \int \frac{x+1}{x^2+2x} dx = \frac{1}{2} \int \frac{2x+2}{x^2+2x} dx = \frac{1}{2} \ln|x^2+2x| + k$$

$$d) \int \frac{5x}{1-x^2} dx = -\frac{5}{2} \int \frac{-2x}{1-x^2} dx = -\frac{5}{2} \ln|1-x^2| + k$$

10. Página 270

$$a) \int \frac{4x^3}{x^4+2} dx = \ln|x^4+2| + k$$

$$b) \int \frac{4x^3}{(x^4+2)^2} dx = \int 4x^3(x^4+2)^{-2} dx = \frac{-1}{x^4+2} + k$$

$$c) \int \frac{4x^3}{\sqrt[3]{x^4+2}} dx = \int 4x^3(x^4+2)^{-\frac{1}{3}} dx = \frac{3}{2} \sqrt[3]{(x^4+2)^2} + k$$

$$d) \int \frac{x-1}{\sqrt{x^2-2x}} dx = \frac{1}{2} \int (2x-2) \cdot (x^2-2x)^{-\frac{1}{2}} dx = \frac{1}{2} \cdot 2 \cdot (x^2-2x)^{\frac{1}{2}} + k = \sqrt{x^2-2x} + k$$

11. Página 271

$$a) \int 3^{\frac{x}{2}} dx = 2 \cdot \int \frac{1}{2} \cdot 3^{\frac{x}{2}} dx = 2 \cdot \frac{3^{\frac{x}{2}}}{\ln 3} + k$$

$$b) \int e^{x+1} dx = e^{x+1} + k$$

$$c) \int \left(\frac{1}{2}\right)^{4x} dx = \frac{1}{4} \int 4 \cdot \left(\frac{1}{2}\right)^{4x} dx = \frac{1}{4} \cdot \frac{\left(\frac{1}{2}\right)^{4x}}{\ln\left(\frac{1}{2}\right)} + k$$

$$d) \int (e^{-3x} + e^{x-2}) dx = \int e^{-3x} dx + \int e^{x-2} dx = -\frac{1}{3} \int -3e^{-3x} dx + \int e^{x-2} dx = -\frac{1}{3} e^{-3x} + e^{x-2} + k$$

12. Página 266

$$a) \int 7^{x^2+1} \cdot 2x dx = \frac{7^{x^2+1}}{\ln(7)} + k$$

$$c) \int \frac{3^{5x-1}}{7} dx = \frac{1}{35} \int 5 \cdot 3^{5x-1} dx = \frac{1}{35} \cdot \frac{3^{5x-1}}{\ln(3)} + k$$

$$b) \int 5e^{\frac{x}{2}+2} dx = 10 \int \frac{1}{2} e^{\frac{x}{2}+2} dx = 10e^{\frac{x}{2}+2} + k$$

$$d) \int \frac{x}{e^{x^2}} dx = -\frac{1}{2} \int -2xe^{-x^2} dx = -\frac{1}{2} e^{-x^2} + k$$

13. Página 272

$$a) \int \operatorname{sen}(2x) dx = \frac{1}{2} \int 2 \cdot \operatorname{sen}(2x) dx = -\frac{1}{2} \cos(2x) + k$$

$$b) \int \cos(x+1) dx = \operatorname{sen}(x+1) + k$$

$$c) \int \frac{\operatorname{sen} \frac{x}{2}}{2} dx = -\cos \frac{x}{2} + k$$

$$d) \int \operatorname{sen}(-x) dx = \cos(-x) + k$$

14. Página 272

$$a) \int \frac{1}{\cos^2(x+1)} dx = \operatorname{tg}(x+1) + k$$

$$b) \int -3 \operatorname{sen}(2x+1) dx = -\frac{3}{2} \int 2 \operatorname{sen}(2x+1) dx = \frac{3}{2} \cos(2x+1) + k$$

$$c) \int (x+1) \cdot \cos(x^2+2x) dx = \frac{1}{2} \int (2x+2) \cdot \cos(x^2+2x) dx = \frac{1}{2} \operatorname{sen}(x^2+2x) + k$$

$$d) \int \frac{x}{\cos^2(x^2-3)} dx = \frac{1}{2} \int \frac{2x}{\cos^2(x^2-3)} dx = \frac{1}{2} \operatorname{tg}(x^2-3) + k$$

15. Página 273

$$a) \int \frac{1}{\sqrt{1-25x^2}} dx = \frac{1}{5} \operatorname{arc} \operatorname{sen}(5x) + k$$

$$b) \int \frac{1}{1+(x-3)^2} dx = \operatorname{arc} \operatorname{tg}(x-3) + k$$

16. Página 273

$$a) \int \frac{1}{\sqrt{1-(2x-3)^2}} dx = \frac{1}{2} \int \frac{2}{\sqrt{1-(2x-3)^2}} dx = \frac{1}{2} \arcsen(2x-3) + k$$

$$b) \int \frac{x}{1+9x^4} dx = \frac{1}{6} \int \frac{6x}{1+(3x^2)^2} dx = \frac{1}{6} \arctg(3x^2) + k$$

17. Página 274

$$a) \int (x^2 + x)e^{-2x+1} dx = (x^2 + x) \left(-\frac{1}{2} e^{-2x+1} \right) + \int \frac{1}{2} e^{-2x+1} \cdot (2x+1) dx =$$

$$u = x^2 + x \rightarrow du = (2x+1) dx$$

$$u = 2x+1 \rightarrow du = 2dx \\ dv = e^{-2x+1} dx \rightarrow v = -\frac{1}{2} e^{-2x+1}$$

$$= (x^2 + x) \left(-\frac{1}{2} e^{-2x+1} \right) + \frac{1}{2} \left[(2x+1) \left(-\frac{1}{2} e^{-2x+1} \right) - \int \left(-\frac{1}{2} e^{-2x+1} \right) \cdot 2 dx \right] =$$

$$= -\frac{1}{2} (x^2 + x) e^{-2x+1} + \frac{1}{2} \left[-\frac{1}{2} (2x+1) e^{-2x+1} - \frac{1}{2} e^{-2x+1} \right] + k =$$

$$= -\frac{1}{2} e^{-2x+1} x^2 - \frac{1}{2} e^{-2x+1} x - \frac{1}{2} e^{-2x+1} x - \frac{1}{4} e^{-2x+1} - \frac{1}{4} e^{-2x+1} + k = -\frac{1}{2} e^{-2x+1} x^2 - e^{-2x+1} x - \frac{1}{2} e^{-2x+1} + k =$$

$$= \left(-\frac{1}{2} x^2 - x - \frac{1}{2} \right) e^{-2x+1} + k$$

$$b) \int x^2 \cdot \cos(3x) dx = x^2 \cdot \frac{1}{3} \text{sen}(3x) - \int \frac{1}{3} \text{sen}(3x) \cdot 2x dx =$$

$$u = x^2 \rightarrow du = 2x dx \\ dv = \cos(3x) dx \rightarrow v = \frac{1}{3} \text{sen}(3x)$$

$$u = x \rightarrow du = dx \\ dv = \text{sen}(3x) dx \rightarrow v = -\frac{1}{3} \text{cos}(3x)$$

$$= \frac{1}{3} x^2 \text{sen}(3x) - \frac{2}{3} \left[-\frac{1}{3} x \cdot \text{cos}(3x) + \frac{1}{3} \int \text{cos}(3x) dx \right] =$$

$$= \frac{1}{3} x^2 \text{sen}(3x) + \frac{2}{9} x \text{cos}(3x) - \frac{2}{27} \text{sen}(3x) + k = \left(\frac{1}{3} x^2 - \frac{2}{27} \right) \text{sen}(3x) + \frac{2}{9} x \text{cos}(3x) + k$$

18. Página 274

$$a) \int 2x^2 \cdot \ln x dx = \frac{2}{3} x^3 \ln x - \int \frac{2}{3} x^3 \cdot \frac{1}{x} dx = \frac{2}{3} x^3 \ln x - \frac{2}{3} \int x^2 dx = \frac{2}{3} x^3 \ln x - \frac{2}{3} \int x^2 dx = \frac{2}{3} x^3 \ln x - \frac{2}{9} x^3 + k$$

$$u = \ln x \rightarrow du = \frac{1}{x} dx \\ dv = 2x^2 dx \rightarrow v = \frac{2}{3} x^3$$

$$b) \int x^2 \cdot 2^x dx = x^2 \cdot \frac{2^x}{\ln 2} - \frac{2}{\ln 2} \int x \cdot 2^x dx = x^2 \cdot \frac{2^x}{\ln 2} - \frac{2}{\ln 2} \left[x \frac{2^x}{\ln 2} - \int \frac{2^x}{\ln 2} dx \right] = \frac{2^x}{\ln 2} x^2 - \frac{2^{x+1}}{(\ln 2)^2} x + \frac{2^{x+1}}{(\ln 2)^3} + k$$

$$u = x^2 \rightarrow du = 2x dx \\ dv = 2^x dx \rightarrow v = \frac{2^x}{\ln 2}$$

$$u = x \rightarrow du = dx \\ dv = 2^x dx \rightarrow v = \frac{2^x}{\ln 2}$$

19. Página 275

$$a) \int \frac{2}{x^2-1} dx = \int \left(\frac{-1}{x+1} + \frac{1}{x-1} \right) dx = \int \frac{-1}{x+1} dx + \int \frac{1}{x-1} dx = -\ln|x+1| + \ln|x-1| + k$$

$$b) \int -\frac{3}{x^2+x-2} dx = \int \frac{1}{x+2} dx + \int \frac{-1}{x-1} dx = \ln|x+2| - \ln|x-1| + k$$

20. Página 275

$$a) \int \frac{2x+1}{x^4-5x^2+4} dx = \int \left(\frac{5}{x-2} + \frac{-1}{x-1} + \frac{-1}{x+1} + \frac{1}{x+2} \right) dx = \frac{5}{12} \int \frac{1}{x-2} dx - \frac{1}{2} \int \frac{1}{x-1} dx - \frac{1}{6} \int \frac{1}{x+1} dx + \frac{1}{4} \int \frac{1}{x+2} dx =$$

$$= \frac{5}{12} \ln|x-2| - \frac{1}{2} \ln|x-1| - \frac{1}{6} \ln|x+1| + \frac{1}{4} \ln|x+2| + k$$

$$b) \int \frac{7x-2}{x^3-2x^2-x+2} dx = \int \left(\frac{4}{x-2} + \frac{-5}{x-1} + \frac{-3}{x+1} \right) dx = 4 \int \frac{1}{x-2} dx - \frac{5}{2} \int \frac{1}{x-1} dx - \frac{3}{2} \int \frac{1}{x+1} dx =$$

$$= 4 \ln|x-2| - \frac{5}{2} \ln|x-1| - \frac{3}{2} \ln|x+1| + k$$

21. Página 276

$$a) \int \frac{x^2}{(x-1)^3} dx = \int \left(\frac{1}{(x-1)^3} + \frac{2}{(x-1)^2} + \frac{1}{x-1} \right) dx = \int \frac{1}{(x-1)^3} dx + \int \frac{2}{(x-1)^2} dx + \int \frac{1}{x-1} dx =$$

$$= \frac{-1}{2(x-1)^2} - \frac{2}{x-1} + \ln|x-1| + k$$

$$b) \int -\frac{3x-2}{(2-x)^2} dx = \int \left(\frac{-4}{(2-x)^2} + \frac{3}{2-x} \right) dx = 4 \int \frac{-1}{(2-x)^2} dx - 3 \int \frac{1}{2-x} dx = \frac{-4}{2-x} - 3 \ln|2-x| + k$$

22. Página 276

$$a) \int \frac{-2x^2+1}{x^3+6x^2+12x+8} dx = \int \left(\frac{-7}{(x+2)^3} + \frac{8}{(x+2)^2} + \frac{-2}{x+2} \right) dx =$$

$$= -7 \int \frac{1}{(x+2)^3} dx + 8 \int \frac{1}{(x+2)^2} dx - 2 \int \frac{1}{x+2} dx = \frac{7}{2(x+2)^2} - \frac{8}{x+2} - 2 \ln|x+2| + k$$

$$b) \int \frac{x-2}{x^4} dx = \int \left(\frac{1}{x^3} + \frac{-2}{x^4} \right) dx = \int \frac{1}{x^3} dx - 2 \int \frac{1}{x^4} dx = \frac{-1}{2x^2} + \frac{2}{3x^3} + k$$

23. Página 277

$$\int \frac{4x^2-2x}{(x+2)(x-3)^2} dx = \int \left(\frac{6}{(x-3)^2} + \frac{16}{x-3} + \frac{4}{x+2} \right) dx =$$

$$= 6 \int \frac{1}{(x-3)^2} dx + \frac{16}{5} \int \frac{1}{x-3} dx + \frac{4}{5} \int \frac{1}{x+2} dx = -\frac{6}{x-3} + \frac{16}{5} \ln|x-3| + \frac{4}{5} \ln|x+2| + k$$

24. Página 277

$$\int \frac{-x^2+7x}{x^3-x^2-x+1} dx = \int \left(\frac{3}{(x-1)^2} + \frac{1}{x-1} + \frac{-2}{x+1} \right) dx = 3 \int \frac{1}{(x-1)^2} dx + \int \frac{1}{x-1} dx - 2 \int \frac{1}{x+1} dx =$$

$$= -\frac{3}{x-1} + \ln|x-1| - 2\ln|x+1| + k$$

25. Página 278

a) $\int \frac{2}{x^2+1} dx = 2\arctg x + k$

b) $\int -\frac{3x-2}{2+x^2} dx = \int -\frac{3x}{2+x^2} dx + \int \frac{2}{2+x^2} dx = -\frac{3}{2} \ln|2+x^2| + \frac{2}{\sqrt{2}} \arctg\left(\frac{x}{\sqrt{2}}\right) + k =$

$$= -\frac{3}{2} \ln|2+x^2| + \sqrt{2} \arctg\left(\frac{x}{\sqrt{2}}\right) + k$$

26. Página 278

a) $\int \frac{-2x^2+1}{x^3-x^2+3x-3} dx = \int \left(\frac{-1}{x-1} + \frac{-7}{x^2+3} \right) dx = \int \frac{-1}{x-1} dx + \int \frac{-7}{x^2+3} dx =$

$$= \int \frac{-1}{x-1} dx + \int \frac{-7}{x^2+3} dx + \int \frac{-7}{x^2+3} dx = -\frac{1}{4} \ln|x-1| - \frac{7}{8} \ln|3+x^2| - \frac{7}{4\sqrt{3}} \arctg\left(\frac{x}{\sqrt{3}}\right) + k$$

b) $\int \frac{x-2}{x^2(x^2+1)} dx = \int \left(\frac{-2}{x^2} + \frac{1}{x} + \frac{-x+2}{x^2+1} \right) dx = \int \frac{-2}{x^2} dx + \int \frac{1}{x} dx + \int \frac{-x+2}{x^2+1} dx =$

$$= \int \frac{-2}{x^2} dx + \int \frac{1}{x} dx + \int \frac{-x}{x^2+1} dx + \int \frac{2}{x^2+1} dx = \frac{2}{x} + \ln|x| - \frac{1}{2} \ln|1+x^2| + 2\arctg x + k$$

27. Página 279

a) $\int \frac{2x^4}{(x-1)^3} dx = \int \left(2x + 6 + \frac{12}{x-1} + \frac{8}{(x-1)^2} + \frac{2}{(x-1)^3} \right) dx = x^2 + 6x + 12\ln|x-1| - \frac{8}{x-1} - \frac{1}{(x-1)^2} + k$

b) $\int -\frac{3x^3-2}{(2-x)^2} dx = \int \left(-3x - 12 + \frac{36}{2-x} - \frac{22}{(2-x)^2} \right) dx = -\frac{3x^2}{2} - 12x - 36\ln|2-x| - \frac{22}{2-x} + k$

28. Página 279

a) $\int \frac{-2x^5+1}{x^4-2x^2+1} dx = \int \left(-2x + \frac{-1}{(x-1)^2} + \frac{-9}{x-1} + \frac{3}{(x+1)^2} + \frac{-7}{x+1} \right) dx =$

$$= \int -2x dx + \int \frac{-1}{(x-1)^2} dx + \int \frac{-9}{x-1} dx + \int \frac{3}{(x+1)^2} dx + \int \frac{-7}{x+1} dx =$$

$$= -x^2 + \frac{1}{4(x-1)} - \frac{9}{4} \ln|x-1| - \frac{3}{4(x+1)} - \frac{7}{4} \ln|x+1| + k$$

b) $\int \frac{x^6-1}{x^2(x^2+1)(x-1)} dx = \int \left(x+1 + \frac{1}{x^2} + \frac{1}{x} + \frac{-x}{x^2+1} + \frac{-1}{x^2+1} \right) dx = \frac{x^2}{2} + x - \frac{1}{x} + \ln|x| - \frac{1}{2} \ln|x^2+1| - \arctg x + k$

29. Página 280

$$a) \int x \cdot 2^{x^2-3} dx = \frac{1}{2} \int 2^t dt = \frac{1}{2} \left(\frac{2^t}{\ln 2} \right) + k = \frac{2^{x^2-3}}{2 \ln 2} + k = \frac{2^{x^2-4}}{\ln 2} + k$$

$$t = x^2 - 3 \rightarrow dt = 2x dx \rightarrow \frac{dt}{2} =$$

$$b) \int \frac{\ln^3 x}{2x} dx = \int \frac{t^3}{2} dt = \frac{1}{2} \frac{t^4}{4} + k = \frac{\ln^4 x}{8} + k$$

$$t = \ln x \rightarrow dt = \frac{1}{x} dx$$

$$dv = \frac{1}{2} dt \rightarrow v = \frac{1}{2} t$$

$$c) \int x \cdot \ln(1+x^2) dx = \int \frac{1}{2} \ln t dt = \frac{1}{2} t \ln t - \int \frac{1}{2} t \cdot \frac{1}{t} dt = \frac{t}{2} \ln t - \frac{t}{2} + k = \frac{1+x^2}{2} \ln(1+x^2) - \frac{1+x^2}{2} + k$$

$$t = 1+x^2 \rightarrow dt = 2x dx \rightarrow \frac{dt}{2} =$$

$$d) \int \frac{1}{e^x + e^{-x}} dx = \int \frac{1}{t + \frac{1}{t}} \cdot \frac{1}{t} dt = \int \frac{1}{t^2 + 1} dt = \arctg(t) + k = \arctg(e^x) + k$$

$$t = e^x \rightarrow dt = e^x dx \rightarrow \frac{dt}{t} =$$

30. Página 280

$$a) \int \frac{x^2+2}{\sqrt{x^3+6x}} dx = \int \frac{1}{3\sqrt{t}} dt = \frac{1}{3} \cdot 2t^{\frac{1}{2}} + k = \frac{2}{3} \sqrt{x^3+6x} + k$$

$$t = x^3 + 6x \rightarrow dt = (3x^2 + 6) dx \rightarrow \frac{dt}{2} =$$

$$b) \int \frac{x}{1+x^4} dx = \frac{1}{2} \int \frac{1}{1+t^2} dt = \frac{1}{2} \arctg(t) + k = \frac{1}{2} \arctg(x^2) + k$$

$$t = x^2 \rightarrow dt = 2x dx \rightarrow \frac{dt}{2} =$$

$$c) \int \frac{\arctg x}{1+x^2} dx = \int t dt = \frac{t^2}{2} + k = \frac{(\arctg x)^2}{2} + k$$

$$t = \arctg x \rightarrow dt = \frac{1}{1+x^2} dx$$

$$d) \int \frac{dx}{(\arcsen x)^5 \sqrt{1-x^2}} = \int \frac{dt}{t^5} = -\frac{1}{4t^4} + k = -\frac{1}{4(\arcsen x)^4} + k$$

$$t = \arcsen x \rightarrow dt = \frac{1}{\sqrt{1-x^2}} dx$$

31. Página 281

$$a) \int \operatorname{sen}^5 x \cos^2 x dx = \int \operatorname{sen} x (1 - \cos^2 x)^2 \cos^2 x dx = - \int (1 - t^2)^2 t^2 dt = - \int (t^6 - 2t^4 + t^2) dt =$$

$$t = \cos x \rightarrow dt = -\operatorname{sen} x dx$$

$$= -\frac{t^7}{7} + \frac{2t^5}{5} - \frac{t^3}{3} + k = -\frac{\cos^7 x}{7} + \frac{2\cos^5 x}{5} - \frac{\cos^3 x}{3} + k$$

$$b) \int \sqrt{4 - x^2} dx = 2 \int \sqrt{1 - \left(\frac{x}{2}\right)^2} dx = 2 \int \sqrt{1 - \operatorname{sen}^2 t} \cdot 2 \cos t dt = 4 \int \cos^2 t dt = 4 \int \left(\frac{1 + \cos 2t}{2}\right) dt$$

$$\operatorname{sen} t = \frac{x}{2} \rightarrow dx = 2 \cos t dt$$

$$= 2t + 2 \operatorname{sen} 2t + k = 2 \operatorname{arc} \operatorname{sen} \frac{x}{2} + 2 \operatorname{sen} 2 \left(\operatorname{arc} \operatorname{sen} \frac{x}{2} \right) + k = 2 \operatorname{arc} \operatorname{sen} \frac{x}{2} + 2 \cdot \frac{x}{2} \sqrt{1 - \left(\frac{x}{2}\right)^2} + k =$$

$$= 2 \operatorname{arc} \operatorname{sen} \frac{x}{2} + \frac{x}{2} \sqrt{4 - x^2} + k$$

32. Página 281

$$a) \int \operatorname{sen}^2 x dx = \int \frac{1 - \cos 2x}{2} dx = \frac{x}{2} - \frac{\operatorname{sen} 2x}{4} + k$$

$$b) \int \frac{\sqrt{2 - x^2}}{4} dx = \frac{\sqrt{2}}{4} \int \sqrt{1 - \left(\frac{x}{\sqrt{2}}\right)^2} dx = \frac{\sqrt{2}}{4} \int \sqrt{1 - \operatorname{sen}^2 t} \sqrt{2} \cos t dt =$$

$$\operatorname{sen} t = \frac{x}{\sqrt{2}} \rightarrow dx = \sqrt{2} \cos t dt$$

$$= \frac{1}{2} \int \cos^2 t dt = \frac{1}{2} \int \frac{1 + \cos 2t}{2} dt = \frac{1}{4} \left(t + \frac{\operatorname{sen} 2t}{2} \right) + k = \frac{1}{4} \operatorname{arc} \operatorname{sen} \frac{x}{\sqrt{2}} + \frac{x}{8} \sqrt{2 - x^2} + k$$

SABER HACER

33. Página 282

$$f(x) = \int \frac{2x}{x^2 + 1} dx = \ln|x^2 + 1| + k$$

$$f(0) = 1 \rightarrow \ln|1| + k = 1 \rightarrow k = 1 \rightarrow f(x) = \ln|x^2 + 1| + 1$$

34. Página 282

$$F(x) = \int \frac{x^2}{\sqrt{1 - x^3}} dx = -\frac{2}{3} \sqrt{1 - x^3} + k$$

$$F(0) = 0 \rightarrow -\frac{2}{3} + k = 0 \rightarrow k = \frac{2}{3} \rightarrow F(x) = -\frac{2}{3} \sqrt{1 - x^3} + \frac{2}{3}$$

35. Página 282

$$\int \frac{dx}{\sqrt{4 - (2x + 1)^2}} = \int \frac{\frac{1}{2}}{\frac{1}{2} \sqrt{4 - (2x + 1)^2}} dx = \frac{1}{2} \int \frac{1}{\sqrt{1 - \left(\frac{2x + 1}{2}\right)^2}} dx = \frac{1}{2} \operatorname{arc} \operatorname{sen} \left(\frac{2x + 1}{2} \right) + k$$

36. Página 282

$$\int e^{2x+e^{2x}} dx = \int t \cdot e^t \cdot \frac{1}{2t} dt = \frac{1}{2} \int e^t dt = \frac{1}{2} e^t + k = \frac{1}{2} e^{e^{2x}} + k$$

$$t = e^{2x} \rightarrow dt = 2e^{2x} dx \rightarrow dx = \frac{dt}{2}$$

37. Página 283

$$x = 5 \operatorname{sen} t \rightarrow dx = 5 \cos t dt$$

$$\int \frac{\sqrt{25-x^2}}{5} dx = \int \frac{\sqrt{25-25\operatorname{sen}^2 t}}{5} \cdot 5 \cos t dt = \int \frac{\sqrt{25(1-\operatorname{sen}^2 t)}}{5} \cdot 5 \cos t dt = 5 \int \sqrt{1-\operatorname{sen}^2 t} \cos t dt =$$

$$= 5 \int \sqrt{\cos^2 t} \cos t dt = 5 \int \cos^2 t dt = 5 \int \frac{1+\cos 2t}{2} dt = \frac{5}{2} \int dt + 5 \int \frac{\cos 2t}{2} dt = \frac{5}{2} t + \frac{5}{4} \operatorname{sen} 2t + k$$

$$x = 5 \operatorname{sen} t \rightarrow t = \operatorname{arc} \operatorname{sen} \left(\frac{x}{5} \right)$$

$$\operatorname{sen} 2t = 2 \operatorname{sen} t \cos t = 2 \left(\frac{x}{5} \right) \sqrt{1 - \left(\frac{x}{5} \right)^2} = \frac{2x}{5} \sqrt{25-x^2}$$

$$\int \frac{\sqrt{25-x^2}}{5} dx = \frac{5}{2} t + \frac{5}{4} \operatorname{sen} 2t + k = \frac{5}{2} \operatorname{arc} \operatorname{sen} \left(\frac{x}{5} \right) + \frac{5}{4} \cdot \frac{2x}{5} \sqrt{25-x^2} + k = \frac{5}{2} \operatorname{arc} \operatorname{sen} \left(\frac{x}{5} \right) + \frac{x}{10} \sqrt{25-x^2} + k$$

38. Página 283

$$I = \int e^x \operatorname{sen} x dx = -e^x \cos x + \int \cos x e^x dx = -e^x \cos x + e^x \operatorname{sen} x - \int e^x \operatorname{sen} x dx = -e^x \cos x + e^x \operatorname{sen} x - I$$

$$u = e^x \rightarrow du = e^x dx$$

$$dv = \operatorname{sen} x dx \rightarrow v = -\cos x$$

Despejamos I:

$$I = -e^x \cos x + e^x \operatorname{sen} x - I \rightarrow I = \frac{e^x (\operatorname{sen} x - \cos x)}{2}$$

Entonces: $\int e^x \operatorname{sen} x dx = \frac{e^x (\operatorname{sen} x - \cos x)}{2} + k$

39. Página 284

$$\int 3 \ln(2x-1) dx = 3x \ln(2x-1) - \int \frac{6x}{2x-1} dx = 3x \ln(2x-1) - \int \left(3 + \frac{3}{2x-1} \right) dx = 3x \ln(2x-1) - 3x - \frac{3}{2} \ln|2x-1| + k$$

$$u = \ln(2x-1) \rightarrow du = \frac{2}{2x-1} dx$$

$$dv = 3 dx \rightarrow v = 3x$$

$$\frac{6x}{2x-1} = 3 + \frac{3}{2x-1}$$

40. Página 284

$$\int (x^2 + 1) \cdot e^{x-2} dx = (x^2 + 1) \cdot e^{x-2} - \int 2x e^{x-2} dx = (x^2 + 1) \cdot e^{x-2} - 2x e^{x-2} + \int 2e^{x-2} dx =$$

$$u = x^2 + 1 \rightarrow du = 2x dx$$

$$dv = e^{x-2} dx \rightarrow v = e^{x-2}$$

$$u = 2x \rightarrow du = 2 dx$$

$$dv = e^{x-2} dx \rightarrow v = e^{x-2}$$

$$= (x^2 + 1) e^{x-2} - 2x e^{x-2} + 2 e^{x-2} + k$$

41. Página 285

$$\int \frac{\sqrt{x+1}+2}{(x+1)^{2/3}-\sqrt{x+1}} dx = \int \frac{\sqrt{t^6+2}}{(t^6)^{2/3}-\sqrt{t^6}} \cdot 6t^5 dt = 6 \int \frac{t^3+2}{t^4-t^3} \cdot t^5 dt =$$

$t^6 = 1+x \rightarrow 6t^5 dt = dx \rightarrow t = (1+x)^{1/6}$

$$= 6 \int \frac{t^5+2t^2}{t-1} dt = 6 \int \left(t^4 + t^3 + t^2 + 3t + 3 + \frac{3}{t-1} \right) dt = \frac{6}{5}t^5 + \frac{3}{2}t^4 + 2t^3 + 9t^2 + 18t + 18 \ln|t-1| + k =$$

$$= \frac{6}{5}(1+x)^{5/6} + \frac{3}{2}(1+x)^{2/3} + 2(1+x)^{1/2} + 9(1+x)^{1/3} + 18(1+x)^{1/6} + 18 \ln|(1+x)^{1/6} - 1| + k$$

42. Página 285

$$\int \frac{1+x}{1+\sqrt{x}} dx = \int \frac{1+t^2}{1+t} \cdot 2t dt = 2 \int \frac{t^3+t}{1+t} dt = 2 \int \left(t^2 - t + 2 - \frac{2}{t+1} \right) dt = \frac{2}{3}t^3 - t^2 + 4t - 4 \ln|t+1| + k =$$

$t = \sqrt{x} \rightarrow dt = \frac{1}{2\sqrt{x}} dx \rightarrow dx = 2t dt$

$$= \frac{2}{3}\sqrt{x^3} - x + 4\sqrt{x} - 4 \ln|\sqrt{x} + 1| + k$$

ACTIVIDADES FINALES

43. Página 286

- a) $F(x) = 3x^5 - 2x + 1 \rightarrow F'(x) = 15x^4 - 2 \rightarrow F(x)$ es primitiva de $f(x)$.
- b) $F(x) = (x^5 - 2)^3 \rightarrow F'(x) = 15x^4(x^5 - 2)^2 \rightarrow F(x)$ es primitiva de $f(x)$.
- c) $F(x) = \frac{2-x}{x^3} + 7 \rightarrow F'(x) = \frac{2x-6}{x^4} \rightarrow F(x)$ es primitiva de $f(x)$.
- d) $F(x) = \frac{x-1}{x^2} - 3 \rightarrow F'(x) = \frac{-x+2}{x^3} \rightarrow F(x)$ es primitiva de $f(x)$.
- e) $F(x) = \ln\sqrt{x^2+2x} \rightarrow F'(x) = \frac{x+1}{x^2+2x} \rightarrow F(x)$ es primitiva de $f(x)$.
- f) $F(x) = 5e^{-x^2} + 11 \rightarrow F'(x) = -10xe^{-x^2} \rightarrow F(x)$ es primitiva de $f(x)$.
- g) $F(x) = \operatorname{arctg}\sqrt{x} \rightarrow F'(x) = \frac{1}{2\sqrt{x}(1+x)} \rightarrow F(x)$ es primitiva de $f(x)$.
- h) $F(x) = \cos^2 x - 1492 \rightarrow F'(x) = 2\operatorname{sen} x \cos x = -\operatorname{sen} 2x \rightarrow F(x)$ es primitiva de $f(x)$.

44. Página 286

- a) $f(x) = 3x^2 \rightarrow F(x) = x^3 + k$ $F(0) = 1 \rightarrow k = 1 \rightarrow F(x) = x^3 + 1$
- b) $f(x) = \frac{4}{5x} \rightarrow F(x) = \frac{4}{5} \ln|x| + k$ $F(1) = 4 \rightarrow k = 4 \rightarrow F(x) = \frac{4}{5} \ln|x| + 4$
- c) $f(x) = \operatorname{sen} 3x \rightarrow F(x) = -\frac{1}{3} \cos 3x + k$ $F(\pi) = -\frac{1}{3} \rightarrow k = -\frac{2}{3} \rightarrow F(x) = -\frac{1}{3} \cos 3x - \frac{2}{3}$
- d) $f(x) = e^{2x} \rightarrow F(x) = \frac{e^{2x}}{2} + k$ $F(0) = \frac{2}{3} \rightarrow k = \frac{1}{6} \rightarrow F(x) = \frac{e^{2x}}{2} + \frac{1}{6}$
- e) $f(x) = \frac{3}{4}(x^2 - 1) \rightarrow F(x) = \frac{x^3}{4} - \frac{3x}{4} + k$ $F(1) = \frac{1}{4} \rightarrow k = \frac{1}{4} + \frac{3}{4} - \frac{1}{4} = \frac{3}{4}$

45. Página 286

a) $F(x) = 1001x + k \rightarrow F'(x) = 1001$

b) $F(x) = x^2 + k \rightarrow F'(x) = 2x$

c) $F(x) = \frac{x^2}{2} + k \rightarrow F'(x) = x$

d) $F(x) = x^3 + k \rightarrow F'(x) = 3x^2$

e) $F(x) = \frac{x^3}{3} + k \rightarrow F'(x) = x^2$

f) $F(x) = x^4 + k \rightarrow F'(x) = 4x^3$

g) $F(x) = x^{n+1} + k \rightarrow F'(x) = (n+1)x^n$

h) $F(x) = \frac{x^{n+1}}{n+1} + k \rightarrow F'(x) = x^n$

46. Página 286

a) $F(x) = \sqrt{x} + k \rightarrow F'(x) = \frac{1}{2\sqrt{x}}$

b) $F(x) = 2\sqrt{x} + k \rightarrow F'(x) = \frac{1}{\sqrt{x}}$

c) $F(x) = 2\sqrt{x+1} + k \rightarrow F'(x) = \frac{1}{\sqrt{x+1}}$

d) $F(x) = 2\sqrt{x-4} + k \rightarrow F'(x) = \frac{1}{\sqrt{x-4}}$

e) $F(x) = \frac{2}{3}\sqrt{3x+10} + k \rightarrow F'(x) = \frac{1}{\sqrt{3x+10}}$

f) $F(x) = \arcsen x + k \rightarrow F'(x) = \frac{1}{\sqrt{1-x^2}}$

47. Página 286

a) $F(x) = 19\ln|x| + k \rightarrow F'(x) = \frac{19}{x}$

b) $F(x) = \frac{\ln|x|}{19} + k \rightarrow F'(x) = \frac{1}{19x}$

c) $F(x) = \ln|19+x| + k \rightarrow F'(x) = \frac{1}{19+x}$

d) $F(x) = 2\ln|2x+3| + k \rightarrow F'(x) = \frac{4}{2x+3}$

e) $F(x) = \frac{1}{x} + k \rightarrow F'(x) = -\frac{1}{x^2}$

f) $F(x) = -\operatorname{arctg} x + k \rightarrow F'(x) = -\frac{1}{1+x^2}$

48. Página 286

a) $F(x) = e^x + k \rightarrow F'(x) = e^x$

b) $F(x) = \frac{e^{2x}}{2} + k \rightarrow F'(x) = e^{2x}$

c) $F(x) = \frac{e^{5x+55}}{5} + k \rightarrow F'(x) = e^{5x+55}$

d) $F(x) = \frac{2^x}{\ln 2} + k \rightarrow F'(x) = 2^x$

e) $F(x) = \frac{2^{x-7}}{\ln 2} + k \rightarrow F'(x) = 2^{x-7}$

f) $F(x) = \frac{2^{9x+5}}{9\ln 2} + k \rightarrow F'(x) = 2^{9x+5}$

49. Página 286

a) $F(x) = \operatorname{sen} x + k \rightarrow F'(x) = \cos x$

b) $F(x) = \frac{\operatorname{sen} 3x}{3} + k \rightarrow F'(x) = \cos 3x$

c) $F(x) = \operatorname{sen}(x+3) + k \rightarrow F'(x) = \cos(x+3)$

d) $F(x) = -3\cos x + k \rightarrow F'(x) = 3\operatorname{sen} x$

e) $F(x) = -\cos(x-\pi) + k \rightarrow F'(x) = \operatorname{sen}(x-\pi)$

f) $F(x) = -3\cos(x-\pi) + k \rightarrow F'(x) = 3\operatorname{sen}(x-\pi)$

50. Página 286

a) Porque la derivada de una función polinómica siempre es otra función polinómica.

b) El grado de $F(x)$ es $n+1$, considerando que $f(x)$ es un polinomio o que n sea distinto de -1 .

51. Página 286

a) $\int (2x - 3)dx = x^2 - 3x + k$

b) $\int (3x^2 + 4x - 2)dx = x^3 + 2x^2 - 2x + k$

c) $\int \left(\frac{3}{4}x^3 - 3x^2 + 6x - 1 \right) dx = \frac{3}{16}x^4 - x^3 + 3x^2 - x + k$

d) $\int (x - 1)^3 dx = \frac{(x - 1)^3}{4} + k$

e) $\int (x + 3)^2 dx = \frac{(x + 3)^3}{3} + k$

f) $\int (1 - 2x)^2 dx = \frac{-1}{2} \int (-2) \cdot (1 - 2x)^2 dx = \frac{-1(1 - 2x)^3}{2 \cdot \frac{3}{-2}} + k = \frac{-1}{6}(1 - 2x)^3 + k$

g) $\int \left(\frac{x^3}{3} - \frac{x^2}{2} + x - 5 \right) dx = \frac{x^4}{12} - \frac{x^3}{6} + \frac{x^2}{2} - 5x + k$

h) $\int (x - 3x + 2) dx = \int (-2x + 2) dx = -x^2 + 2x + k$

52. Página 286

a) $\int 2\sqrt{x} dx = \frac{4}{3}\sqrt{x^3} + k$

b) $\int 2\sqrt{3x} dx = 2\sqrt{3} \int x^{\frac{1}{2}} dx = \frac{4\sqrt{3}}{3}x^{\frac{3}{2}} + k = \frac{4}{3}\sqrt{3x^3} + k$

c) $\int \left(\frac{1}{\sqrt{x}} - x \right) dx = 2\sqrt{x} - \frac{x^2}{2} + k$

d) $\int (\sqrt[4]{x} + \sqrt[3]{x}) dx = \frac{4}{5}\sqrt[4]{x^5} + \frac{3}{4}\sqrt[3]{x^4} + k$

e) $\int (2 + \sqrt{x}) dx = 2x + \frac{2}{3}\sqrt{x^3} + k$

f) $\int (2 + \sqrt{3x}) dx = 2x + \frac{2}{3}\sqrt{3x^3} + k$

g) $\int \left(\frac{1}{\sqrt[3]{x}} - x \right) dx = \frac{3}{2}\sqrt[3]{x^2} - \frac{x^2}{2} + k$

h) $\int \left(\frac{2}{\sqrt{2x}} + \frac{4}{\sqrt[4]{4x}} \right) dx = \sqrt{2} \int x^{-\frac{1}{2}} dx + 2\sqrt{2} \int x^{-\frac{1}{4}} dx = 2\sqrt{2}x^{\frac{1}{2}} + \frac{8\sqrt{2}}{3}x^{\frac{3}{4}} + k = 2\sqrt{2x} + \frac{8\sqrt{2}}{3}\sqrt[4]{x^3} + k$

53. Página 286

a) $\int \frac{4}{x-2} dx = 4\ln|x+2| + k$

d) $\int \frac{1}{(x+4)^2} dx = -\frac{1}{x-4} + k$

b) $\int \left(\frac{1}{x} + \frac{2}{x^2} + \frac{3}{x^3} \right) dx = \ln|x| - \frac{2}{x} - \frac{3}{2x^2} + k$

e) $\int \left(\frac{1}{(x-1)^2} + \frac{7}{x+3} \right) dx = \frac{-1}{x-1} + 7\ln|x+3| + k$

c) $\int \left(\frac{2}{x+3} - \frac{1}{x-4} \right) dx = 2\ln|x+3| - \ln|x-4| + k$

f) $\int \left(\frac{2}{(x+3)^3} + \frac{5}{(x-3)^2} \right) dx = \frac{-1}{(x+3)^2} - \frac{5}{(x-3)} + k$

54. Página 287

a) $\int e^{x-2} dx = e^{x-2} + k$

b) $\int (e^x + 1) dx = e^x + x + k$

c) $\int (e^{2x} + 2^x) dx = \frac{1}{2}e^{2x} + \frac{2^x}{\ln 2} + k$

d) $\int \left(xe^{x^2} + \frac{4}{3}x^3\right) dx = \frac{1}{2}e^{x^2} + \frac{1}{3}x^4 + k$

e) $\int e^{2x+3} dx = \frac{1}{2}e^{2x+3} + k$

f) $\int (2e^x - 3x^2) dx = 2e^x - x^3 + k$

g) $\int \left(5e^{\frac{x}{2}} + 2 \cdot 3^x\right) dx = 10e^{\frac{x}{2}} + 2 \frac{3^x}{\ln 3} + k$

h) $\int (x2^{x^2+2x} + 2^{x^2+2x}) dx = \frac{1}{2} \int (2x+2) \cdot 2^{x^2+2x} dx = \frac{1}{2} \cdot \frac{2^{x^2+2x}}{\ln 2} + k$

55. Página 287

a) $\int \cos(2x) dx = \frac{1}{2} \operatorname{sen}(2x) + k$

b) $\int 4 \operatorname{sen}(x + \pi) dx = -4 \cos(x + \pi) + k$

c) $\int 3 \cos\left(\frac{\pi}{2} - \frac{x}{3}\right) dx = -9 \operatorname{sen}\left(\frac{\pi}{2} - \frac{x}{3}\right) + k$

d) $\int 5 \operatorname{sen}(2x - \pi) dx = \frac{-5}{2} \cos(2x - \pi) + k$

e) $\int 3 \sec^2\left(\frac{1}{5}x\right) dx = 15 \int \frac{\frac{1}{5} dx}{\cos^2\left(\frac{1}{5}x\right)} = 15 \operatorname{tg}\left(\frac{1}{5}x\right) + k$

f) $\int \frac{7}{\operatorname{sen}^2(3x)} dx = \frac{7}{3} \int \frac{3}{\operatorname{sen}^2(3x)} dx = -\frac{7}{3} \operatorname{cotg}(3x) + k$

g) $\int \frac{5}{\cos^2\left(\frac{x}{3}\right)} dx = 15 \int \frac{\left(\frac{1}{3}\right)}{\cos^2\left(\frac{x}{3}\right)} dx = 15 \operatorname{tg}\left(\frac{x}{3}\right) + k$

h) $\int \frac{3}{\sqrt{1-x^2}} dx = 3 \operatorname{arc} \operatorname{sen} x + k$

i) $\int \frac{3}{x^2+1} dx = 3 \operatorname{arctg} x + k$

j) $\int \frac{1}{(3x)^2+1} dx = \frac{1}{3} \operatorname{arctg}(3x) + k$

56. Página 287

a) Consideramos $\int x^2 dx$ y comprobamos que no coincide con el producto $\int x dx \cdot \int x dx$.

$$\int x^2 dx = \frac{x^3}{3} + k$$

$$\int x dx = \frac{x^2}{2} \rightarrow \int x dx \cdot \int x dx = \frac{x^4}{4} + k$$

Entonces la afirmación es cierta.

b) Consideramos $\int 1 dx$ y comprobamos que no coincide con $\frac{\int x dx}{\int x dx}$.

$$\int 1 dx = \int \frac{x}{x} dx = x + k$$

$$\int x dx = \frac{x^2}{2} \rightarrow \frac{\int x dx}{\int x dx} = \frac{\frac{x^2}{2}}{\frac{x^2}{2}} + k = 1 + k$$

Entonces la afirmación es cierta.

57. Página 287

- a) $\int \frac{2x}{x^2+1} dx = \ln|x^2+1| + k$
- b) $\int \frac{8x-3}{4x^2-3x+1} dx = \ln|4x^2-3x+1| + k$
- c) $\int \frac{6x^2+1}{2x^3+x-9} dx = \ln|2x^3+x-9| + k$
- d) $\int \frac{\operatorname{sen} x}{\operatorname{cos} x} dx = -\ln|\operatorname{cos} x| + k$
- e) $\int \operatorname{cot} x dx = \int \frac{\operatorname{cos} x}{\operatorname{sen} x} dx = \ln|\operatorname{sen} x| + k$
- f) $\int 3x^2 \operatorname{sen} x^3 dx = -\operatorname{cos} x^3 + k$
- g) $\int (2x+1)\operatorname{sen}(x^2+x+5) dx = -\operatorname{cos}(x^2+x+5) + k$
- h) $\int 6x \operatorname{cos}(3x^2-5) dx = \operatorname{sen}(3x^2-5) + k$
- i) $\int \left(\frac{1}{x^2+1} + \frac{1}{x^2} \right) dx = \operatorname{arctg} x - \frac{1}{x} + k$
- j) $\int 6xe^{3x^2} dx = e^{3x^2} + k$
- k) $\int (3x^2+1)e^{x^3+x} dx = e^{x^3+x} + k$
- l) $\int (12x^2-6x)e^{4x^3-3x^2+7} dx = e^{4x^3-3x^2+7} + k$
- m) $\int xe^{7x^2} dx = \frac{e^{7x^2}}{14} + k$
- n) $\int \frac{-2}{4+x^2} dx = \frac{-1}{2} \int \frac{1}{1+\left(\frac{x}{2}\right)^2} dx = -\operatorname{arctg} \frac{x}{2} + k$
- ñ) $\int \frac{-2}{\sqrt{3-x^2}} dx = \frac{-2}{\sqrt{3}} \int \frac{1}{\sqrt{1-\left(\frac{x}{\sqrt{3}}\right)^2}} dx = -2 \operatorname{arcsen} \frac{x}{\sqrt{3}} + k$
- o) $\int x^3 e^{x^2} = \frac{1}{2} \int te^t dt = \frac{1}{2} e^{x^2} (x^2-1) + k$ (con $t = x^2$ y $dt = 2x dx$)
- p) $\int \frac{x+\ln x}{x} dx = \int dx + \int \frac{\ln x}{x} dx = x + \int \frac{\ln x}{x} dx = x + \frac{\ln^2 x}{2} + k$
- q) $\int \left(\frac{1}{x^2+1} - \frac{2}{x^2} + \frac{3}{x} \right) dx = \operatorname{arctg} x + \frac{2}{x} + 3 \ln|x| + k$

58. Página 287

- a) $\int \frac{x}{x^2-3} dx = \frac{1}{2} \int \frac{2x}{x^2-3} dx = \frac{1}{2} \ln|x^2-3| + k$
- b) $\int \frac{x^3}{\sqrt{x^4+3}} dx = \frac{1}{4} \int \frac{1}{\sqrt{t}} dt = \frac{1}{2} \sqrt{t} + k = \frac{1}{2} \sqrt{x^4+3} + k$
 $t = x^4 + 3 \rightarrow dt = 4x^3 dx \rightarrow x^3 dx = \frac{1}{4} dt$
- c) $\int \frac{\operatorname{sen} x}{1-\operatorname{cos} x} dx = \ln|1-\operatorname{cos} x| + k$
- d) $\int \frac{3+x}{\sqrt{1-x^2}} dx = \int \frac{3}{\sqrt{1-x^2}} dx + \int \frac{x}{\sqrt{1-x^2}} dx =$
 $3 \operatorname{arcsen} x - \frac{1}{2} \int \frac{1}{\sqrt{t}} dt = 3 \operatorname{arcsen} x - \sqrt{1-x^2} + k$
 $t = 1-x^2 \rightarrow dt = -2x dx \rightarrow x dx = \frac{1}{2} dt$
- e) $\int \frac{2x+\sqrt{x}}{x^2} dx = \int \frac{2x}{x^2} dx + \int \frac{\sqrt{x}}{x^2} dx =$
 $= \int \frac{2x}{x^2} dx + \int x^{-\frac{3}{2}} dx = \ln|x^2| - \frac{2}{\sqrt{x}} + k$
- f) $\int \frac{2 \operatorname{cos} x}{3+\operatorname{sen} x} dx = 2 \ln|3+\operatorname{sen} x| + k$
- g) $\int \frac{x}{x^2+2^4} dx = \frac{1}{2} \int \frac{dt}{t} = \frac{1}{2} \ln|x^2+2^4| + k$
 $t = x^2 + 2^4 \rightarrow dt = 2x dx$
- h) $\int \frac{x}{\sqrt{1+3x^2}} dx = \frac{1}{6} \int \frac{1}{\sqrt{t}} dt = \frac{2}{6} \sqrt{t} + k = \frac{1}{3} \sqrt{1+3x^2} + k$
 $t = 1+3x^2 \rightarrow dt = 6x dx \rightarrow x dx = \frac{1}{6} dt$
- i) $\int \frac{x-1}{x^2+1} dx = \int \frac{x}{x^2+1} dx + \int \frac{-1}{x^2+1} dx =$
 $= \frac{1}{2} \ln|x^2+1| - \operatorname{arctg} x + k$
- j) $\int \frac{8}{x^2+4} dx = \int \frac{2}{\left(\frac{x}{2}\right)^2+1} dx = 4 \int \frac{\frac{1}{2}}{\left(\frac{x}{2}\right)^2+1} dx =$
 $= 4 \operatorname{arctg} \left(\frac{x}{2} \right) + k$

59. Página 287

$$F(x) = \int (x+1)(x^2+2x+6) dx = \int (x^3+3x^2+8x+6) dx = \frac{x^4}{4} + x^3 + 4x^2 + 6x + k$$

$$F(0) = 1 \rightarrow k = 1 \rightarrow F(x) = \frac{x^4}{4} + x^3 + 4x^2 + 6x + 1$$

60. Página 287

$$f(x) = \int \frac{a}{1+x} dx = a \ln|1+x| + k$$

$$f(0) = 1 \rightarrow a \ln 1 + k \rightarrow k = 1 \qquad f(1) = -1 \rightarrow a \ln 2 + 1 = -1 \rightarrow a = \frac{-2}{\ln 2}$$

La función es: $f(x) = \frac{-2}{\ln 2} \ln|1+x| + 1$

61. Página 287

$$F(x) = \int \frac{-x}{1-x^2} dx = \frac{1}{2} \int \frac{-2x}{1-x^2} dx = \frac{1}{2} \ln|1-x^2| + k$$

$$F(\sqrt{2}) = 3 \rightarrow \frac{1}{2} \ln|1-2| + k = 3 \rightarrow k = 3$$

La función primitiva es: $F(x) = \frac{1}{2} \ln|1-x^2| + 3$

62. Página 287

Sabemos que f pasa por el origen de coordenadas, por lo que: $f(0) = 0$

Además, ese es un punto de inflexión, entonces: $f''(0) = 0$

Como la pendiente de la recta tangente en $(0, 0)$ es 5, podemos concluir que: $f'(0) = 5$

Finalmente, tenemos:

$$f'''(x) = 24x - 6$$

$$f''(x) = \int (24x - 6) dx = 12x^2 - 6x + k \rightarrow f''(0) = k = 0 \rightarrow f''(x) = 12x^2 - 6x$$

$$f'(x) = \int (12x^2 - 6x) dx = 4x^3 - 3x^2 + k \rightarrow f'(0) = 5 \rightarrow k = 5 \rightarrow f'(x) = 4x^3 - 3x^2 + 5$$

$$f(x) = \int (4x^3 - 3x^2 + 5) dx = x^4 - x^3 + 5x + k \rightarrow f(0) = 0 \rightarrow k = 0 \rightarrow f(x) = x^4 - x^3 + 5x$$

63. Página 287

$$f''(x) = \int (x+1) dx = \frac{x^2}{2} + x + k \rightarrow f''(0) = 1 \rightarrow k = 1 \rightarrow f''(x) = \frac{x^2}{2} + x + 1$$

$$f'(x) = \int \left(\frac{x^2}{2} + x + 1 \right) dx = \frac{x^3}{6} + \frac{x^2}{2} + x + k \rightarrow f'(0) = 5 \rightarrow k = 5 \rightarrow f'(x) = \frac{x^3}{6} + \frac{x^2}{2} + x + 5$$

$$f(x) = \int \left(\frac{x^3}{6} + \frac{x^2}{2} + x + 5 \right) dx = \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + 5x + k \rightarrow f(0) = 0 \rightarrow k = 0 \rightarrow f(x) = \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + 5x$$

64. Página 287

$$F(x) = \int \frac{1 - \operatorname{sen} x}{2x + 2\cos x} dx = \frac{1}{2} \int \frac{1 - \operatorname{sen} x}{x + \cos x} dx = \frac{1}{2} \ln|x + \cos x| + k$$

$$F(0) = 2 \rightarrow 0 + k = 2 \rightarrow k = 2 \rightarrow F(x) = \frac{1}{2} \ln|x + \cos x| + 2$$

65. Página 287

$$f'(x) = \int \operatorname{sen} x dx = -\cos x + k$$

$$f(x) = \int (-\cos x + k) dx = -\operatorname{sen} x + kx + l$$

$$f(0) = 1 \rightarrow l = 1 \rightarrow f(x) = -\operatorname{sen} x + kx + 1$$

$$f\left(\frac{\pi}{2}\right) = \pi \rightarrow -1 + \frac{\pi}{2}k + 1 = \pi \rightarrow k = 2 \rightarrow f(x) = -\operatorname{sen} x + 2x + 1$$

66. Página 287

$$f'(x) = \int (6x + 2) dx = 3x^2 + 2x + k$$

f tiene un mínimo relativo en $A(1, 3)$; por tanto, $f(1) = 3$ y $f'(1) = 0$.

$$f'(1) = 0 \rightarrow 3 + 2 + k = 0 \rightarrow k = -5 \rightarrow f'(x) = 3x^2 + 2x - 5$$

$$f(x) = \int (3x^2 + 2x - 5) dx = x^3 + x^2 - 5x + k \rightarrow f(1) = 3 \rightarrow 1 + 1 - 5 + k = 3 \rightarrow k = 6 \rightarrow f(x) = x^3 + x^2 - 5x + 6$$

67. Página 288

a) $F(x) = \int \frac{3x^2 + \cos x + 2 \cdot e^{2x}}{x^3 + \operatorname{sen} x + e^{2x}} dx = \ln|x^3 + \operatorname{sen} x + e^{2x}| + k \rightarrow F(0) = -5 \rightarrow k = -5$

$$F(x) = \ln|x^3 + \operatorname{sen} x + e^{2x}| - 5$$

b) $F(0) = k$ Basta con tomar $k = 0$. Entonces $F(x) = \ln|x^3 + \operatorname{sen} x + e^{2x}|$ pasa por el origen de coordenadas.

c) $F(0) = k$ Basta con tomar $k = 1$. Entonces $F(x) = \ln|x^3 + \operatorname{sen} x + e^{2x}| + 1$ pasa por el punto $B(0, 1)$.

68. Página 288

a) Tenemos que $f(-1) = -4$.
$$f(x) = \begin{cases} f_1(x) & \text{si } x \leq 1 \\ f_2(x) & \text{si } x > 1 \end{cases}$$

$$f_1(x) = \int (2 - x) dx = 2x - \frac{x^2}{2} + k \rightarrow f_1(-1) = -4 \rightarrow -2 - \frac{1}{2} + k = -4 \rightarrow k = -\frac{3}{2}$$

$$f_1(x) = 2x - \frac{x^2}{2} - \frac{3}{2} \quad \text{si } x \leq 1 \quad f_2(x) = \int \frac{1}{x} dx = \ln|x| + k \quad \text{si } x > 1$$

Como f es derivable, entonces es continua; por tanto: $f_1(1) = f_2(1) \rightarrow 2 - \frac{1}{2} - \frac{3}{2} = 0 = k \rightarrow f_2(x) = \ln|x|$

$$\text{Entonces: } f(x) = \begin{cases} f_1(x) = 2x - \frac{x^2}{2} - \frac{3}{2} & \text{si } x \leq 1 \\ f_2(x) = \ln|x| & \text{si } x > 1 \end{cases}$$

b) La pendiente de la recta tangente en el punto $x = 2$ es $f'(2) = \frac{1}{2}$. Además, $f(2) = \ln 2$. Por tanto, la ecuación de la

$$\text{recta es: } y = \frac{1}{2}x + n \rightarrow \ln 2 = \frac{1}{2} \cdot 2 + n \rightarrow n = \ln 2 - 1 \rightarrow y = \frac{1}{2}x + \ln 2 - 1$$

69. Página 288

$$a) \int x^3 \ln x dx = \frac{x^4 \ln x}{4} - \int \frac{x^3}{4} dx = \frac{x^4 \ln x}{4} - \frac{x^4}{16} + k = \frac{x^4}{4} \left(\ln x - \frac{1}{4} \right) + k$$

$$u = \ln x \rightarrow du = \frac{1}{x} dx$$

$$b) \int \ln(2x+1) dx = x \ln|2x+1| - \int \frac{2x}{2x-1} dx = x \ln|2x+1| - \int \left(1 - \frac{1}{2x+1} \right) dx =$$

$$u = \ln(2x+1) \rightarrow du = \frac{2}{2x+1} dx$$

$$= x \ln|2x+1| - x + \frac{\ln|2x+1|}{2} + k = \left(x + \frac{1}{2} \right) \ln|2x+1| - x + k$$

$$c) I = \int e^{-x} \sin 2x dx = -e^{-x} \frac{\cos 2x}{2} - \frac{1}{2} \int e^{-x} \cos 2x dx = -e^{-x} \frac{\cos 2x}{2} - \frac{1}{2} \left(e^{-x} \frac{\sin 2x}{2} + \frac{1}{2} \int e^{-x} \sin 2x dx \right) =$$

$$u = e^{-x} \rightarrow du = -e^{-x} dx$$

$$dv = \sin 2x dx \rightarrow v = -\frac{\cos 2x}{2}$$

$$u = e^{-x} \rightarrow du = -e^{-x} dx$$

$$dv = \cos 2x dx \rightarrow v = \frac{\sin 2x}{2}$$

$$= -e^{-x} \frac{\cos 2x}{2} - e^{-x} \frac{\sin 2x}{4} - \frac{1}{4} \int e^{-x} \sin 2x dx = -e^{-x} \frac{\cos 2x}{2} - e^{-x} \frac{\sin 2x}{4} - \frac{1}{4} I$$

$$I = -e^{-x} \frac{\cos 2x}{2} - e^{-x} \frac{\sin 2x}{4} - \frac{1}{4} I \rightarrow \frac{5}{4} I = -e^{-x} \left(\frac{\cos 2x}{2} + \frac{\sin 2x}{4} \right) \rightarrow I = \frac{-e^{-x}}{5} (2 \cos 2x + \sin 2x) + k$$

$$d) \int \arctg x dx = x \cdot \arctg x - \int \frac{x}{x^2+1} dx = x \cdot \arctg x - \frac{\ln|x^2+1|}{2} + k$$

$$u = \arctg x \rightarrow du = \frac{1}{x^2+1} dx$$

$$e) I = \int \frac{\ln x}{x} dx = \ln^2 x - \int \frac{\ln x}{x} dx = \ln^2 x - I \rightarrow I = \ln^2 x - I \rightarrow I = \int \frac{\ln x}{x} dx = \frac{1}{2} \ln^2 x + k$$

$$u = \ln x \rightarrow du = \frac{1}{x} dx$$

$$f) \int x \sin 2x dx = -\frac{x \cos 2x}{2} + \int \frac{\cos 2x}{2} dx = -\frac{x \cos 2x}{2} + \frac{\sin 2x}{4} + k$$

$$u = x \rightarrow du = dx$$

$$g) \int x^2 \sin 2x dx = \frac{-x^2 \cos 2x}{2} + \int x \cos 2x dx = \frac{-x^2 \cos 2x}{2} + \frac{x \sin 2x}{2} - \int \frac{\sin 2x}{2} dx =$$

$$u = x^2 \rightarrow du = 2x dx$$

$$dv = \sin 2x dx \rightarrow v = -\frac{\cos 2x}{2}$$

$$u = x \rightarrow du = dx$$

$$dv = \cos 2x dx \rightarrow v = \frac{\sin 2x}{2}$$

$$= \frac{-x^2 \cos 2x}{2} + \frac{x \sin 2x}{2} + \frac{\cos 2x}{4} + k$$

$$h) \int (2x+3)e^{2x} dx = \frac{(2x+3)e^{2x}}{2} - \int e^{2x} dx = \frac{(2x+3)e^{2x}}{2} - \frac{e^{2x}}{2} + k = (x+1)e^{2x} + k$$

$$u = 2x+3 \rightarrow du = 2x dx$$

$$i) \int \frac{x}{e^x} dx = -\frac{x}{e^x} + \int e^{-x} dx = -\frac{x}{e^x} - \frac{1}{e^x} + k = \frac{-x-1}{e^x} + k$$

$$u = x \rightarrow du = dx$$

$$j) \int (x^2 - 5) \cos x dx = (x^2 - 5) \operatorname{sen} x - 2 \int x \operatorname{sen} x dx = (x^2 - 5) \operatorname{sen} x + 2x \cos x - 2 \int \cos x dx =$$

$$u = x^2 - 5 \rightarrow du = 2x dx$$

$$u = x \rightarrow du = dx$$

$$dv = \operatorname{sen} x dx \rightarrow v = -\cos$$

$$= (x^2 - 5) \operatorname{sen} x + 2x \cos x - 2 \operatorname{sen} x + k = (x^2 - 7) \operatorname{sen} x + 2x \cos x + k$$

$$k) \int (2x^2 + x - 2)e^{3x} dx = \frac{(2x^2 + x - 2)e^{3x}}{3} - \int \frac{(4x + 1)e^{3x}}{3} dx = \frac{(2x^2 + x - 2)e^{3x}}{3} - \frac{1}{3} \left(\frac{(4x + 1)e^{3x}}{3} - \int \frac{4e^{3x}}{3} dx \right) =$$

$$u = 2x^2 + x - 2 \rightarrow du = (4x + 1) dx$$

$$u = 4x + 1 \rightarrow du = 4 dx$$

$$= \frac{(2x^2 + x - 2)e^{3x}}{3} - \frac{(4x + 1)e^{3x}}{9} + \frac{4}{27} e^{3x} + k = \frac{1}{27} e^{3x} (18x^2 - 3x - 17) + k$$

$$l) \int (2 + e^{2x}) \cos(x + 1) dx = \int 2 \cos(x + 1) dx + \int e^{2x} \cos(x + 1) dx = 2 \operatorname{sen}(x + 1) + e^{2x} \operatorname{sen}(x + 1) - \int 2e^{2x} \operatorname{sen}(x + 1) dx =$$

$$u = e^{2x} \rightarrow du = 2e^{2x} dx$$

$$dv = \cos(x + 1) dx \rightarrow v = \operatorname{sen}(x)$$

$$u = e^{2x} \rightarrow du = 2e^{2x} dx$$

$$dv = \operatorname{sen}(x + 1) dx \rightarrow v = -\cos$$

$$= 2 \operatorname{sen}(x + 1) + e^{2x} \operatorname{sen}(x + 1) + 2e^{2x} \cos(x + 1) - 2 \int 2e^{2x} \cos(x + 1) dx = 2 \operatorname{sen}(x + 1) + \frac{e^{2x} \operatorname{sen}(x + 1) + 2e^{2x} \cos(x + 1)}{5} + k$$

70. Página 288

$$a) \int \ln x^2 dx = x \ln x^2 - \int x \cdot \frac{2x}{x^2} dx = x \ln x^2 - \int 2 dx = x \ln x^2 - 2x + k$$

$$u = \ln x^2 \rightarrow du = \frac{2x}{x^2} dx$$

$$b) \int \ln x^3 dx = x \ln x^3 - \int x \cdot \frac{3x^2}{x^3} dx = x \ln x^3 - \int 3 dx = x \ln x^3 - 3x + k$$

$$u = \ln x^3 \rightarrow du = \frac{3x^2}{x^3} dx$$

$$c) \int \ln x^4 dx = x \ln x^4 - \int x \cdot \frac{4x^3}{x^4} dx = x \ln x^4 - \int 4 dx = x \ln x^4 - 4x + k$$

$$u = \ln x^4 \rightarrow du = \frac{4x^3}{x^4} dx$$

$$d) \int \ln x^5 dx = x \ln x^5 - \int x \cdot \frac{5x^4}{x^5} dx = x \ln x^5 - \int 5 dx = x \ln x^5 - 5x + k$$

$$u = \ln x^5 \rightarrow du = \frac{5x^4}{x^5} dx$$

$$e) \int \ln x^n dx = x \ln x^n - \int x \cdot \frac{nx^{n-1}}{x^n} dx = x \ln x^n - \int n dx = x \ln x^n - nx + k$$

$$u = \ln x^n \rightarrow du = \frac{nx^{n-1}}{x^n} dx$$

71. Página 288

$$a) \int x^2 \cdot e^{3x} dx = \frac{1}{3} x^2 e^{3x} - \frac{2}{3} \int x e^{3x} dx = \frac{1}{3} x^2 e^{3x} - \frac{2}{3} \left[\frac{1}{3} x e^{3x} - \frac{1}{3} \int e^{3x} dx \right] = \frac{1}{3} x^2 e^{3x} - \frac{2}{9} x e^{3x} + \frac{2}{9} \cdot \frac{1}{3} e^{3x} + k =$$

$$\boxed{\begin{array}{l} u = x^2 \rightarrow du = 2x dx \\ u = x \rightarrow du = dx \\ dv = e^{3x} dx \rightarrow v = \frac{1}{3} e^{3x} \end{array}}$$

$$= \frac{e^{3x}}{3} \left(x^2 - \frac{2}{3} x + \frac{2}{9} \right) + k$$

$$b) \int \ln \left(\frac{x+1}{x-1} \right) dx = \int x \ln \left(\frac{x+1}{x-1} \right) dx = \int x \ln(x+1) dx - \int x \ln(x-1) dx = \frac{x}{x+1} - \int \frac{1}{x+1} dx - \frac{x}{x-1} + \int \frac{1}{x-1} dx =$$

$$\boxed{\begin{array}{l} u = x \rightarrow du = dx \\ dv = \ln(x+1) dx \rightarrow v = \frac{1}{2} (x+1)^2 \end{array}}$$

$$\boxed{\begin{array}{l} u = x \rightarrow du = dx \\ dv = \ln(x-1) dx \rightarrow v = \frac{1}{2} (x-1)^2 \end{array}}$$

$$= \frac{x}{x+1} - \ln|x+1| - \frac{x}{x-1} + \ln|x-1| + k$$

$$c) \int \frac{\ln x}{x^3} dx = -\frac{\ln x}{2x^2} + \frac{1}{2} \int \frac{1}{x^3} dx = -\frac{\ln x}{2x^2} - \frac{1}{4x^2} + k$$

$$\boxed{u = \ln x \rightarrow du = \frac{1}{x} dx}$$

$$d) \int x \cdot \operatorname{arctg}(x+1) dx = \frac{1}{2} x^2 \operatorname{arctg}(x+1) - \int \frac{x^2}{2(x^2+2x+2)} dx = \frac{1}{2} x^2 \operatorname{arctg}(x+1) - \frac{1}{2} \int \left(1 - \frac{2(x+1)}{x^2+2x+2} \right) dx =$$

$$\boxed{u = \operatorname{arctg}(x+1) \rightarrow du = \frac{1}{1+(x+1)^2} dx}$$

$$= \frac{1}{2} x^2 \operatorname{arctg}(x+1) - \frac{1}{2} \int dx + \frac{1}{2} \int \frac{2(x+1)}{x^2+2x+2} dx = \frac{1}{2} x^2 \operatorname{arctg}(x+1) - \frac{1}{2} x + \frac{1}{2} \ln|x^2+2x+2| + k$$

$$e) \int x \cdot 2^x dx = x \cdot \frac{2^x}{\ln 2} - \int \frac{2^x}{\ln 2} dx = x \cdot \frac{2^x}{\ln 2} - \frac{1}{\ln 2} \int 2^x dx = x \cdot \frac{2^x}{\ln 2} - \frac{1}{\ln 2} \cdot \frac{1}{\ln 2} \int \ln 2 \cdot 2^x dx = \frac{2^x x}{\ln 2} - \frac{2^x}{\ln^2 2} + k$$

$$\boxed{u = x \rightarrow du = dx}$$

$$f) \int x \cdot \ln^2 x dx = \ln^2 x \cdot \frac{x^2}{2} - \int x \ln x dx = \ln^2 x \cdot \frac{x^2}{2} - \left(\frac{x^2}{2} \ln x - \frac{1}{2} \int x dx \right) = \ln^2 x \cdot \frac{x^2}{2} - \frac{x^2}{2} \ln x - \frac{1}{4} x^2 + k =$$

$$\boxed{\begin{array}{l} u = \ln^2 x \rightarrow du = \frac{2 \ln x}{x} dx \\ u = \ln x \rightarrow du = \frac{1}{x} dx \end{array}}$$

$$= \frac{1}{2} x^2 \left(\ln^2 x - \ln x - \frac{1}{4} \right) + k$$

$$g) \int \operatorname{arc sen} x dx = x \operatorname{arc sen} x - \int \frac{x}{\sqrt{1-x^2}} dx = x \operatorname{arc sen} x + \frac{1}{2} \int \frac{-2x}{\sqrt{1-x^2}} dx = x \operatorname{arc sen} x + \sqrt{1-x^2} + k$$

$$\boxed{u = \operatorname{arc sen} x \rightarrow du = \frac{1}{\sqrt{1-x^2}} dx}$$

$$h) \int \frac{x \cdot \operatorname{arc sen} x}{\sqrt{1-x^2}} dx = \int t \operatorname{sen} t dt = -t \operatorname{cost} + \int \operatorname{cost} dt = -t \operatorname{cost} + \operatorname{sent} = -\cos(\operatorname{arc sen} x) \cdot \operatorname{arc sen} x + x + k =$$

$$\boxed{\begin{array}{l} t = \operatorname{arc sen} x \rightarrow dt = \frac{1}{\sqrt{1-x^2}} dx \\ u = t \rightarrow du = dt \end{array}}$$

$$= -\operatorname{arc sen} x \sqrt{1-x^2} + x + k$$