72. Página 288

a) $\int \sqrt[3]{x}\ln x\,dx = \frac{3}{4}\ln x\sqrt[3]{x^4} - \frac{3}{4}\int \sqrt[3]{x}\,dx = \frac{3}{4}\ln x\sqrt[3]{x^4} - \frac{3}{4}\cdot\frac{3}{4}\sqrt[3]{x^4} = \frac{3}{4}\ln x\sqrt[3]{x^4} - \frac{9}{16}\sqrt[3]{x^4}$

$u = \ln x \to du = \frac{1}{x}dx$

b) $\int (x^2+3x)e^{-x+7}\,dx = -(x^2+3x)e^{-x+7} + \int (2x+3)e^{-x+7}\,dx = -(x^2+3x)e^{-x+7} - (2x+3)e^{-x+7} - 2e^{-x+7} + k$

$u = x^2+3x \to du = (2x+3$

$u = 2x+3 \to du = 2dx$
$dv = e^{-x+7}\,dx \to v = -e^{-x+7}$

c) $\int e^x\cos(3x)dx = \frac{1}{3}e^x sen3x - \frac{1}{3}\int e^x sen3x\,dx = \frac{1}{3}e^x\,sen3x - \frac{1}{3}\left(-\frac{1}{3}\cos 3x + \frac{1}{3}\int e^x\cos 3x\,dx\right) =$

$u = e^x \to du = e^x dx$
$dv = \cos 3x\,dx \to v = \frac{1}{3}sen$

$u = e^x \to du = e^x dx$
$dv = sen3x\,dx \to v = -\frac{1}{3}c$

$= \frac{1}{3}e^x sen3x + \frac{1}{9}\cos 3x - \frac{1}{9}\int e^x\cos 3x\,dx$

$I = \frac{1}{3}e^x sen3x + \frac{1}{9}\cos 3x - \frac{1}{9}I \to I = \frac{9}{10}\left(\frac{1}{3}e^x sen3x + \frac{1}{9}\cos 3x\right) \to I = \frac{3}{10}e^x sen3x + \frac{1}{10}\cos 3x$

d) $\int \ln(1+x^2)dx = x\ln(1+x^2) - \int \frac{2x^2}{x^2+1}dx = x\ln(1+x^2) - \left[\int\left(2 - \frac{2}{x^2+1}\right)dx\right] = x\ln(1+x^2) - 2x - 2arc\,tg\,x + k$

$u = \ln(1+x^2) \to du = \frac{2x}{1+x^2}$

e) $\int x^2\cdot arc\,tg\,x\,dx = \frac{x^3}{3}arc\,tg\,x - \int\frac{x^3}{3(x^2+1)}dx = \frac{x^3}{3}arc\,tg\,x - \frac{1}{3}\int\left(x - \frac{x}{x^2+1}\right)dx = \frac{x^3}{3}arc\,tg\,x - \frac{1}{6}x^2 + \frac{1}{6}\ln|x^2+1| + k$

$u = arc\,tg\,x \to du = \frac{1}{x^2+1}d$

f) $\int \frac{\sqrt{x}+1}{\sqrt[3]{x}}\ln x\,dx = \int\left(x^{\frac{1}{6}} + x^{-\frac{1}{3}}\right)\ln x\,dx = \ln x\left(\frac{6}{7}x^{7/6} + \frac{3}{2}x^{2/3}\right) - \int\frac{\frac{6}{7}x^{7/6} + \frac{3}{2}x^{2/3}}{x}dx =$

$u = \ln x \to du = \frac{1}{x}dx$
$dv = (x^{1/6} + x^{-1/3})dx \to v = \frac{6}{7}x^{7/6}$

$= \ln x\left(\frac{6}{7}x^{7/6} + \frac{3}{2}x^{2/3}\right) - \int\left(\frac{6}{7}x^{1/6} + \frac{3}{2}x^{-1/3}\right)dx = \left(\frac{6}{7}x^{7/6} + \frac{3}{2}x^{2/3}\right)\ln x - \frac{36}{49}x^{7/6} - \frac{9}{4}x^{2/3} + k$

73. Página 288

$\int x\cdot sen(2x+1)\cdot\cos(2x-1)dx = \frac{1}{2}\int x(sen(2) + sen(4x))dx = \frac{1}{2}sen(2)\int x dx + \frac{1}{2}\int x\,sen(4x)dx =$

$= \frac{1}{4}sen(2)x^2 + \frac{1}{2}\left(-\frac{1}{4}x\cos(4x) + \frac{1}{4}\int\cos(4x)dx\right) = \frac{1}{4}x^2 sen(2) - \frac{1}{8}x\cos(4x) + \frac{1}{32}sen(4x) + k$

$u = x \to du = dx$
$dv = sen4x\,dx \to v = -\frac{1}{4}\cos 4x$

74. Página 288

$$f(x)=\int x^2\cdot sen\,x\,dx=-x^2\cos x+2\int x\cos x\,dx=-x^2\cos x+2\left(x\,sen\,x-\int sen\,x\,dx\right)=$$

$u=x^2\to du=2x\,dx$
$dv=sen\,x\,dx\to v=-c$

$u=x\to du=dx$
$dv=\cos x\,dx\to v=se$

$$=-x^2\cos x+2x\,sen\,x+2\cos x+k$$

$$f(0)=1\to 2\cos(0)+k=1\to k=1-2=-1$$

$$f(x)=-x^2\cos x+2x\,sen\,x+2\cos x-1$$

75. Página 288

$$f'(x)=\int f''(x)dx=\int 2x\cdot\ln x dx=2\left(\frac{1}{2}x^2\ln x-\int\frac{1}{2}x\,dx\right)=x^2\ln x-\frac{1}{2}x^2+k$$

$u=\ln x\to du=\frac{1}{x}dx$

$$f'(1)=0\to\ln(1)-\frac{1}{2}+k=0\to k=k=\frac{1}{2}\qquad f'(x)=x^2\ln x-\frac{1}{2}x^2+\frac{1}{2}$$

$$f(x)=\int f'(x)=\int\left(x^2\ln x-\frac{1}{2}x^2+\frac{1}{2}\right)dx=\int x^2\ln x dx-\frac{1}{2}\int x^2dx+\frac{1}{2}\int 1dx=\frac{1}{3}x^3\ln x-\frac{1}{3}\int x^2dx-\frac{1}{6}x^3+\frac{1}{2}x=$$

$u=\ln x\to du=\frac{1}{x}dx$

$$=\frac{1}{3}x^3\ln x-\frac{1}{9}x^3-\frac{1}{6}x^3+\frac{1}{2}x+k=\left(\frac{1}{3}\ln x-\frac{5}{18}\right)x^3+\frac{1}{2}x+k$$

$$f(e)=\frac{e}{2}\to\left(\frac{1}{3}-\frac{5}{18}\right)e^3+\frac{1}{2}e+k=\frac{e}{2}\to k=-\frac{1}{18}e^3$$

$$f(x)=\left(\frac{1}{3}\ln x-\frac{5}{18}\right)x^3+\frac{1}{2}x-\frac{1}{18}e^3$$

76. Página 288

$$F(x)=\int f(x)dx=\int(ax^2+x\cdot\cos x+1)dx=a\int x^2\,dx+\int x\cos x\,dx+\int dx=\frac{a}{3}x^3+x\,sen\,x-\int sen\,x\,dx+x=$$

$u=x\to du=dx$
$dv=\cos x\,dx\to v=sen\,x$

$$=\frac{a}{3}x^3+x sen x+\cos x+x+k$$

Tomamos $k=0$: $F(\pi)=\pi\to\frac{a}{3}\pi^3-1+\pi=\pi\to a=\frac{3}{\pi^3}$

77. Página 288

a) $f(x)=\int f'(x)dx=\int x^2\,sen\,x\,dx=-x^2\cos x+2\int x\cos x\,dx=-x^2\cos x+2x\,sen\,x+2\cos x+k=$

$u=x^2\to du=2xdx$
$dv=sen\,x\,dx\to v=-c$

$u=x\to du=dx$
$dv=\cos x\,dx\to v=se$

$$=(2-x^2)\cos x+2x\,sen\,x+k$$

$$f(0)=1\to 2+k=1\to k=-3\to f(x)=(2-x^2)\cos x+2x\,sen\,x-3$$

b) $f(x)=\int f'(x)dx=\int x\ln x\,dx=\frac{x^2\ln x}{2}-\int\frac{x}{2}dx=\frac{x^2\ln x}{2}-\frac{x^2}{4}+k$

$u=\ln x \to du=\frac{1}{x}dx$

$f(1)=\frac{1}{2}\to-\frac{1}{4}+k=\frac{1}{2}\to k=\frac{3}{4}\to f(x)=\frac{x^2\ln x}{2}-\frac{x^2}{4}+\frac{3}{4}$

78. Página 288

La pendiente de la recta tangente es el valor de la derivada de la función.

$$F(x)=\int(2x+3)e^{2x}\,dx=\frac{(2x+3)e^{2x}}{2}-\int e^{2x}\,dx=\frac{(2x+3)e^{2x}}{2}-\frac{e^{2x}}{2}+k=$$

$$=\frac{(2x+3)e^{2x}-e^{2x}}{2}+k=(x+1)e^{2x}+k \;\to\; F(0)=1\to 1+k=1\to k=0\to F(x)=(x+1)e^{2x}$$

79. Página 288

$$F(x)=\int f(x)dx=\int\frac{3}{1+x^2}dx=3\,arctg(x)+k$$

$$F(1)=\frac{\pi}{2}\to 3\,arctg(1)+k=\frac{\pi}{2}\to k=\frac{\pi}{2}-\frac{3\pi}{4}\to k=-\frac{\pi}{4}\to F(x)=3\,arctg(x)-\frac{\pi}{4}$$

80. Página 288

a) $\int\frac{3}{x^2-3x+2}dx=\int\left(\frac{-3}{x-1}+\frac{3}{x-2}\right)dx=\int\frac{-3}{x-1}dx+\int\frac{3}{x-2}dx=-3\ln|x-1|+3\ln|x-2|+k$

b) $\int\frac{2-x}{x^2+3x}dx=\int\left(\frac{\frac{2}{3}}{x}+\frac{\frac{-5}{3}}{x+3}\right)dx=\frac{2}{3}\int\frac{1}{x}dx+\frac{-5}{3}\int\frac{1}{x+3}dx=\frac{2}{3}\ln|x|-\frac{5}{3}\ln|x+3|+k$

c) $\int\frac{x-3}{x^2-4}dx=\int\left(\frac{\frac{5}{4}}{x+2}+\frac{\frac{-1}{4}}{x-2}\right)dx=\frac{5}{4}\int\frac{1}{x+2}dx-\frac{1}{4}\int\frac{1}{x-2}dx=\frac{5}{4}\ln|x+2|-\frac{1}{4}\ln|x-2|+k$

d) $\int\frac{x}{x^2+6x+5}dx=\int\left(\frac{-\frac{1}{4}}{x+1}+\frac{\frac{5}{4}}{x+5}\right)dx=-\frac{1}{4}\int\frac{1}{x+1}dx+\frac{5}{4}\int\frac{1}{x+5}dx=-\frac{1}{4}\ln|x+1|+\frac{5}{4}\ln|x+5|+k$

81. Página 288

a) $\int f(x)dx=\int\frac{3}{x^2-2x+1}dx=3\int\frac{1}{(x-1)^2}dx=\frac{-3}{x-1}+k$

b) $\int g(x)dx=\int\frac{x+2}{x^2-2x+1}dx=\int\left(\frac{1}{(x-1)}+\frac{3}{(x-1)^2}\right)dx=\int\frac{1}{(x-1)}dx+\int\frac{3}{(x-1)^2}dx=\ln|x-1|-\frac{3}{x-1}+k$

c) $\int h(x)dx=\int\frac{x}{x^2-2x+1}dx=\int\left(\frac{1}{(x-1)}+\frac{1}{(x-1)^2}\right)dx=\int\frac{1}{(x-1)}dx+\int\frac{1}{(x-1)^2}dx=\ln|x-1|-\frac{1}{x-1}+k$

d) $\int i(x)dx=\int\frac{x^2}{x^2-2x+1}dx=\int\left(1+\frac{2x-1}{x^2-2x+1}\right)dx=\int 1dx+\int\frac{2x-1}{x^2-2x+1}dx=x+\int\left(\frac{2}{x-1}+\frac{1}{(x-1)^2}\right)dx=$

$=x+2\ln|x-1|-\frac{1}{x-1}+k$

82. Página 289

a) $\int \frac{2}{x^3+x^2}dx = \int\left(-\frac{2}{x}+\frac{2}{x^2}+\frac{2}{x+1}\right)dx = -2\ln|x| - \frac{2}{x} + 2\ln|x+1| + k$

b) $\int \frac{1+x^2}{3x^2-x^3}dx = \int\left(\frac{\frac{1}{9}}{x}+\frac{\frac{1}{3}}{x^2}+\frac{\frac{10}{9}}{3-x}\right)dx = \frac{1}{9}\ln|x| - \frac{1}{3x} - \frac{10}{9}\ln|3-x| + k$

c) $\int \frac{x+2}{x^3-x^2-x+1}dx = \int\left(\frac{-\frac{1}{4}}{x-1}+\frac{\frac{3}{2}}{(x-1)^2}+\frac{\frac{1}{4}}{x+1}\right)dx = -\frac{1}{4}\ln|x-1| - \frac{3}{2}\cdot\frac{1}{x-1} + \frac{1}{4}\ln|x+1| + k$

d) $\int \frac{1+x^3}{x^3-2x^2}dx = \int\left(1+\frac{2x^2+1}{x^3-2x^2}\right)dx = \int 1dx + \int\left(\frac{\frac{-1}{4}}{x}+\frac{\frac{-1}{2}}{x^2}+\frac{\frac{9}{4}}{x-2}\right)dx = x - \frac{1}{4}|\ln x| + \frac{1}{2x} + \frac{9}{4}\ln|x-2| + k$

83. Página 289

a) $\int \frac{2}{x^4-x^2}dx = \int\left(\frac{-2}{x^2}+\frac{-1}{x+1}+\frac{1}{x-1}\right)dx = \frac{2}{x} - \ln|x+1| + \ln|x-1| + k$

b) $\int \frac{x+1}{4x^2-x^4}dx = \int\left(\frac{1}{4x}+\frac{1}{4x^2}+\frac{3}{16(2-x)}-\frac{1}{16(2+x)}\right)dx = \frac{1}{4}\ln|x| - \frac{1}{4x} - \frac{3}{16}\ln|2-x| - \frac{1}{16}\ln|2+x| + k$

c) $\int \frac{x+6}{x^4-3x^3+x^2+3x-2}dx = \int\left(-\frac{9}{4(x-1)}-\frac{5}{12(x+1)}-\frac{7}{2(x-1)^2}+\frac{8}{3(x-2)}\right)dx =$

$= -\frac{9}{4}\ln|x-1| - \frac{5}{12}\ln|x+1| + \frac{7}{2(x-1)} + \frac{8}{3}\ln|x-2| + k$

84. Página 289

a) $\int \frac{2}{x^3+x}dx = \int\left(\frac{2}{x}-\frac{2x}{x^2+1}\right)dx = 2\ln|x| - \ln|x^2+1| + k$

b) $\int \frac{x+2}{x^3+x^2+x+1}dx = \int\left(\frac{3}{2(x^2+1)}-\frac{x}{2(x^2+1)}+\frac{1}{2(x+1)}\right)dx = \frac{3}{2}arctgx - \frac{1}{4}\ln|x^2+1| + \frac{1}{2}\ln|x+1| + k$

c) $\int \frac{x+3}{4x^3-4x^2+x-1}dx = \int\left(\frac{-16x-11}{5(4x^2+1)}+\frac{4}{5(x-1)}\right)dx = \int\left(\frac{-16x}{5(4x^2+1)}-\frac{11}{5(4x^2+1)}+\frac{4}{5(x-1)}\right)dx =$

$= -\frac{2}{5}\ln|4x^2+1| - \frac{11}{10}arctg(2x) + \frac{4}{5}\ln|x-1| + k$

d) $\int \frac{1}{(x-2)^2(x^2+2)}dx = \int\left(\frac{2x}{18(x^2+2)}+\frac{1}{18(x^2+2)}-\frac{1}{9(x-2)}+\frac{1}{6(x-2)^2}\right)dx =$

$= \frac{1}{18}\ln|x^2+2| - \frac{1}{6(x-2)} - \frac{1}{9}\ln|x-2| + \frac{\sqrt{2}}{36}arctg\left(\frac{x}{\sqrt{2}}\right) + k$

e) $\int \frac{3x^2+1}{x^4-1}dx = \int\left(\frac{1}{x^2+1}-\frac{1}{x+1}+\frac{1}{x-1}\right)dx = \ln|1-x| - \ln|x+1| + arctg\,x + k$

f) $\int \frac{x^2+x+1}{x^3-x^2-x+1}dx = \int\left(\frac{3}{4(x-1)}+\frac{3}{2(x-1)^2}+\frac{1}{4(x+1)}\right)dx = \frac{3}{4}\ln|x-1| - \frac{3}{2(x-1)} + \frac{1}{4}\ln|x+1| + k$

g) $\int \frac{x^2+1}{(x-1)^3}dx = \int\left(\frac{1}{x-1}+\frac{2}{(x-1)^2}+\frac{2}{(x-1)^3}\right)dx = \ln|x-1| - \frac{2}{(x-1)} - \frac{1}{(x-1)^2} + k = \ln|x-1| + \frac{1-2x}{(x-1)^2} + k$

85. Página 289

a) $\int \frac{x+3}{x-1}dx = \int \left(1+\frac{4}{x-1}\right)dx = x+4\ln|x-1|+k$

b) $\int \frac{2x+3}{2x+1}dx = \int \left(1+\frac{2}{2x+1}\right)dx = x+\ln|2x+1|+k$

c) $\int \frac{3}{x^2+x-2}dx = \int \left(\frac{1}{x-1}-\frac{1}{x+2}\right)dx = \ln|x-1|-\ln|x+2|+k$

d) $\int \frac{5x-1}{x^2-1}dx = \int \left(\frac{2}{x-1}+\frac{3}{x+1}\right)dx = 2\ln|x-1|+3\ln|x+1|+k$

e) $\int \frac{x-2}{x^2-x}dx = \int \left(\frac{-1}{x-1}+\frac{2}{x}\right)dx = -\ln|x-1|+2\ln|x|+k$

f) $\int \frac{x+2}{x^2-1}dx = \int \left(\frac{3}{2(x-1)}-\frac{1}{2(x+1)}\right)dx = \frac{3}{2}\ln|x-1|-\frac{1}{2}\ln|x+1|+k$

g) $\int \frac{1}{x^2+9}dx = \frac{1}{3}arc\,tg\frac{x}{3}+k$

h) $\int \frac{12}{x^3+4x^2+x-6}dx = \int \left(\frac{1}{x-1}-\frac{4}{x+2}+\frac{3}{x+3}\right)dx = \ln|x-1|-4\ln|x+2|+3\ln|x+3|+k$

86. Página 289

a) $\int \frac{1}{x^2-5x+6}dx = \int \left(\frac{1}{x-3}-\frac{1}{x-2}\right)dx = \ln|x-3|-\ln|x-2|+k$

b) $\int \frac{x-2}{x^2+2x-3}dx = \int \frac{1}{4}\left(\frac{5}{x+3}-\frac{1}{x-1}\right)dx = \frac{5}{4}\ln|x+3|-\frac{1}{4}\ln|x-1|+k$

c) $\int \frac{x-4}{x^2+2x-3}dx = \int \frac{1}{4}\left(\frac{7}{x+3}-\frac{3}{x-1}\right)dx = \frac{7}{4}\ln|x+3|-\frac{3}{4}\ln|x-1|+k$

d) $\int \frac{2x+8}{x^2-4}dx = \int \left(\frac{3}{x-2}-\frac{1}{x+2}\right)dx = 3\ln|x-2|-\ln|x+2|+k$

e) $\int \frac{x^2}{x-4}dx = \int \left(x+4+\frac{16}{x-4}\right)dx = \frac{x^2}{2}+4x+16\ln|x-4|+k$

f) $\int \frac{x}{x^4+1}dx = \frac{1}{2}arc\,tg\,x^2+k$

g) $\int \frac{x^4-x^3-x+1}{x^3-x^2}dx = \int \left(x-\frac{1}{x^2}\right)dx = \frac{x^2}{2}+\frac{1}{x}+k$

h) $\int \frac{x^3}{(x+1)^4}dx = \int \left(\frac{-1}{(x+1)^4}+\frac{3}{(x+1)^3}-\frac{3}{(x+1)^2}+\frac{1}{x+1}\right)dx =$

$=\frac{1}{3(x+1)^3}-\frac{3}{2(x+1)^2}+\frac{3}{x+1}+\ln|x+1|+k$

i) $\int \frac{2x+5}{(x+3)^3}dx = \int \left(\frac{-1}{(x+3)^3}+\frac{2}{(x+3)^2}\right)dx = \frac{1}{2(x+3)^2}-\frac{2}{x+3}+k$

j) $\int \frac{2x}{(x+1)^2}dx = \int \left(\frac{-2}{(x+1)^2}+\frac{2}{x+1}\right)dx = \frac{2}{x+1}+2\ln|x+1|+k$

87. Página 289

a) $\int \frac{4x^3+2x-1}{2x+1}dx = \int\left(2x^2-x-\frac{5}{2(2x+1)}+\frac{3}{2}\right)dx = \frac{2}{3}x^3-\frac{1}{2}x^2-\frac{5}{4}\ln|2x+1|+\frac{3}{2}x+k$

b) $\int \frac{-x^2+x-1}{3-x}dx = \int\left(x+2-\frac{7}{3-x}\right)dx = \frac{1}{2}x^2+2x+7\ln|3-x|+k$

c) $\int \frac{x^3}{x^2+4x-5}dx = \int\left(x+\frac{1}{6(x-1)}+\frac{125}{6(x+5)}-4\right)dx = \frac{1}{2}(x-8)x+\frac{1}{6}\ln|1-x|+\frac{125}{6}\ln|x+5|+k$

d) $\int \frac{x^3-x+6}{x^2+5x+4}dx = \int\left(x+\frac{2}{x+1}+\frac{18}{x+4}-5\right)dx = \frac{1}{2}x^2+2\ln|x+1|+18\ln|x+4|-5x+k$

88. Página 289

a) $\int x^2\cdot\sqrt{x^3+3}\,dx = \frac{1}{3}\int\sqrt{t}dt = \frac{1}{3}\cdot\frac{2}{3}\sqrt{t^3}+k = \frac{2}{9}\sqrt{t^3}+k = \frac{2}{9}\sqrt{(x^3+3)^3}+k$

$t = x^3+3 \rightarrow dt = 3x^2dx \rightarrow \frac{1}{3x^2}dt = dx$

b) $\int x^3\cdot e^{x^4+1}dx = \int\frac{1}{4}e^t dt = \frac{1}{4}e^t+k = \frac{1}{4}e^{x^4+1}+k$

$t = x^4+1 \rightarrow dt = 4x^3dx \rightarrow \frac{1}{4}dt = x^3dx$

c) $\int \frac{2}{x\cdot\ln x}dx = 2\int\frac{1}{t}dt = 2\ln|t|+k = 2\ln|\ln x|+k$

$t = \ln x \rightarrow dt = \frac{1}{x}dx$

d) $\int \frac{\ln x^2}{x}dx = \int\frac{2\ln x}{x}dx = 2\int\frac{\ln x}{x}dx = 2\int t\,dt = 2\frac{t^2}{2}+k = \ln^2 x+k$

$t = \ln x \rightarrow dt = \frac{1}{x}dx$

89. Página 289

Hacemos en todos los apartados el cambio de variable:

$t = \sqrt{x} \rightarrow dt = \frac{1}{2\sqrt{x}}dx \rightarrow 2dt = \frac{1}{\sqrt{x}}dx$

a) $\int \frac{e^{\sqrt{x}+1}}{\sqrt{x}}dx = 2\int e^{t+1}dt = 2e^{t+1}+k = 2e^{\sqrt{x}+1}+k$

b) $\int \frac{sen\sqrt{x}}{\sqrt{x}}dx = 2\int sen t\,dt = -2cos\,t+k = -2cos(\sqrt{x})+k$

c) $\int \frac{e^{-\sqrt{x}+1}}{\sqrt{x}}dx = 2\int e^{-t+1}dt = -2e^{-t+1}+k = -2e^{-\sqrt{x}+1}+k$

d) $\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}dx = 2\int\sqrt{1+t}\,dt = 2\int(1+t)^{\frac{1}{2}}dt = \frac{4\sqrt{(1+t)^3}}{3}+k = \frac{4\sqrt{(1+\sqrt{x})^3}}{3}+k$

90. Página 289

a) $\int \cos x \, sen^3 x \, dx = \int t^3 \, dt = \frac{t^4}{4} + k = \frac{sen^4 x}{4} + k$

$t = sen\, x \rightarrow dt = \cos x \, dx$

b) $\int x \ln(1+x^2) dx = \frac{1}{2}\int \ln t \, dt = \frac{1}{2}(t \ln t - t) + k = \frac{1}{2}\left((1+x^2)\ln(1+x^2) - (1+x^2)\right) + k$

$t = 1 + x^2 \rightarrow dt = 2x dx$

c) $\int \frac{\ln 2x}{x} dx = \int t \, dt = \frac{t^2}{2} + k = \frac{\ln^2(2x)}{2} + k$

$t = \ln 2x \rightarrow dt = \frac{1}{x} dx$

d) $\int 2x \, sen x^2 \, dx = \int sen\, t \, dt = -\cos t + k = -\cos x^2 + k$

$t = x^2 \rightarrow dt = 2x \, dx$

e) $\int x\sqrt{x+1} dx = \int 2(t^2-1)t^2 \, dt = \frac{2}{5}t^5 - \frac{2}{3}t^3 + k = \sqrt{x+1}\left(\frac{2(x+1)^2}{5} - \frac{2(x+1)}{3}\right) + k$

$t = \sqrt{x+1} \rightarrow t^2 = x+1 \rightarrow 2t \, dt = dx$

f) $\int \frac{2x}{x^2-1} dx = \int \frac{1}{t} dt = \ln|t| + k = \ln|x^2-1| + k$

$t = x^2 - 1 \rightarrow dt = 2x \, dx$

g) $\int \cos^2 x \, sen\, x \, dx = \int -t^2 \, dt = \frac{-t^3}{3} + k = \frac{-\cos^3 x}{3} + k$

$t = \cos x \rightarrow dt = -sen\, x \, dx$

h) $\int sen\, x \, e^{\cos x} dx = \int -e^t \, dt = -e^t + k = -e^{\cos x} + k$

$t = \cos x \rightarrow dt = -sen\, x \, dx$

i) $\int \cos^2 x \, sen^3 x \, dx = \int -t^2(1-t^2) dt = -\frac{t^3}{3} + \frac{t^5}{5} + k = -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + k$

$t = \cos x \rightarrow dt = -sen\, x \, dx$

j) $\int \frac{sen^3 x}{\cos x} dx = \int \frac{t^2-1}{t} dt = \frac{t^2}{2} - \ln|t| + k = \frac{\cos^2 x}{2} - \ln|\cos x| + k$

$t = \cos x \rightarrow dt = -sen\, x \, dx$

k) $\int (x^2+1)e^{x^3+3x} dx = \int \frac{e^t}{3} dt = \frac{e^t}{3} + k = \frac{e^{x^3+3x}}{3} + k$

$t = x^3 + 3x \rightarrow dt = (3x^2+3) dx$

l) $\int \cos^5 x \, sen^3 x \, dx = \int t^5(t^2-1) dt = \frac{t^8}{8} - \frac{t^6}{6} + k = \frac{\cos^8 x}{8} - \frac{\cos^6 x}{6} + k$

$t = \cos x \rightarrow dt = -sen\, x \, dx$

91. Página 289

a) $\int \frac{2}{4+x^2}dx = \int \frac{1}{1+t^2}dt = \text{arc tg}\, t + k = \text{arc tg}\frac{x}{2} + k$

$t = \frac{x}{2} \rightarrow dt = \frac{1}{2}dx$

b) $\int x(x+5)^{10}dx = \int (t-5)t^{10}dt = \int (t^{11} - 5t^{10})dt = \frac{t^{12}}{12} - \frac{5t^{11}}{11} + k = \frac{(x+5)^{12}}{12} - \frac{5(x+5)^{11}}{11} + k$

$t = x + 5 \rightarrow dt = dx$

c) $\int \frac{tg\sqrt{x}}{\sqrt{x}}dx = \int 2tg\, t\, dt = -2\ln|\cos t| + k = -2\ln|\cos\sqrt{x}| + k$

$t = \sqrt{x} \rightarrow dt = \frac{1}{2\sqrt{x}}dx$

d) $\int xe^{3x^2}dx = \int \frac{e^t}{6}dt = \frac{e^t}{6} + k = \frac{e^{3x^2}}{6} + k$

$t = 3x^2 \rightarrow dt = 6x\, dx$

e) $\int \frac{x}{\sqrt{1-x^4}}dx = \frac{1}{2}\int \frac{1}{\sqrt{1-t^2}}dt = \frac{1}{2}\text{arc sen}\, t + k = \frac{1}{2}\text{arc sen}\, x^2 + k$

$t = x^2 \rightarrow dt = 2x\, dx$

f) $\int \frac{x}{\sqrt{1-x^2}}dx = \int \frac{-1}{2\sqrt{t}}dt = -\sqrt{t} + k = -\sqrt{1-x^2} + k$

$t = 1 - x^2 \rightarrow dt = -2x\, dx$

g) $\int tg\, 2x\, dx = \int \frac{-1}{2t}dt = \frac{-1}{2}\ln|t| + k = \frac{-1}{2}\ln|\cos 2x| + k$

$t = \cos 2x \rightarrow dt = -2\,\text{sen}\, 2x\, dx$

h) $\int \text{cotg}\frac{x}{5}dx = \int 5t\, dt = 5\ln|t| + k = 5\ln|\text{sen}\frac{x}{5}| + k$

$t = \text{sen}\frac{x}{5} \rightarrow dt = \frac{1}{5}\cos\frac{x}{5}dx$

i) $\int \frac{x^4}{\sqrt{1-x^{10}}}dx = \frac{1}{5}\int \frac{1}{\sqrt{1-t^2}}dt = \frac{1}{5}\text{arcsen}\, t + k = \frac{1}{5}\text{arcsen}\, x^5 + k$

$t = x^5 \rightarrow dt = 5x^4\, dx$

j) $\int e^x\sqrt{(e^x+1)^3}dx = \int \sqrt{t^3}dt + k = \frac{2}{5}\sqrt{(e^x+1)^5} + k$

$t = e^x + 1 \rightarrow dt = e^x dx$

92. Página 290

a) $\int \cos^3 x\, \text{sen}^5 x\, dx = \int \cos x(1-\text{sen}^2 x)\text{sen}^5 x\, dx = \int (1-t^2)t^5 dt = \int t^5 dt - \int t^7 dt = \frac{1}{6}t^6 - \frac{1}{8}t^8 + k =$

$= \frac{1}{6}\text{sen}^6 x - \frac{1}{8}\text{sen}^8 x + k$

$t = \text{sen}\, x \rightarrow dt = \cos x\, dx$

b) $\int \frac{\text{sen}^3 x}{\cos^2 x}dx = \int \frac{(1-\cos^2 x)\text{sen}\, x}{\cos^2 x}dx = \int \frac{t^2-1}{t^2}dt = \int dt - \int \frac{1}{t^2}dt = t + \frac{1}{t} + k = \cos x + \frac{1}{\cos x} + k$

$t = \cos x \rightarrow dt = -\text{sen}\, x\, dx$

c) $\int sen^3 x\cos^{15} x\,dx = \int (1-\cos^2 x) sen\,x\cos^{15} x\,dx = \int (t^2-1)t^{15}\,dt = \int t^{17}\,dt - \int t^{15}\,dt = \frac{1}{18}t^{18} - \frac{1}{16}t^{16} + k =$

$t = \cos x \to dt = -sen\,x\,dx$

$= \frac{1}{18}\cos^{18} x - \frac{1}{16}\cos^{16} x + k$

$t = \cos x \to dt = -sen\,x\,dx$

d) $\int \frac{1}{\cos^3 x\cdot sen\,x}dx = -\int \frac{-sen\,x}{\cos^3 x(1-\cos^2 x)} = -\int \frac{1}{(1-t^2)t^3}dt = \int\left(-\frac{1}{t} - \frac{1}{t^3} + \frac{1}{2(t+1)} + \frac{1}{2(t-1)}\right)dt =$

$= -\ln|t| + \frac{1}{2t^2} + \frac{1}{2}\ln|t+1| + \frac{1}{2}\ln|t-1| + k = -\ln|\cos x| + \frac{1}{2\cos^2 x} + \frac{1}{2}\ln|\cos x+1| + \frac{1}{2}\ln|\cos x-1| + k =$

$= \ln\left|\frac{\sqrt{|1-\cos^2 t|}}{\cos t}\right| + \frac{1}{2\cos^2 t} + k = \ln|tg\,t| + \frac{1}{2\cos^2 t} + k$

93. Página 290

a) $\int \cos^3 dx = \int (1-sen^2 x)\cos x\,dx = \int (1-t^2)dt = t + \frac{t^3}{3} + k = sen\,x + \frac{sen^3 x}{3} + k$

$t = sen\,x \to dt = \cos x\,dx$

$t = \sec x \to dt = tg\,x\sec x\,dx$

b) $\int tg^3 x\cdot\sec^3 x\,dx = \int tg\,x(\sec^2 x - 1)\sec^3 x\,dx = \int (t^2-1)t^2 dt = \int (t^4 - t^2)dt = \frac{1}{5}t^5 - \frac{1}{3}t^3 + k = \frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + k$

c) $\int sen^5 x\,dx = \int sen\,x(1-\cos^2 x)^2 dx = \int -\left(1-t^2\right)^2 dt = \int (-1+2t^2-t^4)dt = \frac{-t^5}{5} + \frac{2t^3}{3} - t + k =$

$t = \cos x \to dt = -sen\,x\,dx$

$= \frac{-\cos^5 x}{5} + \frac{2\cos^3 x}{3} - \cos x + k$

d) $\int \frac{\cos x}{sen x + \cos x}dx = \int \frac{\cos x\cdot\sec^3 x}{(sen\,x+\cos x)\cdot\sec^3 x}dx = \int \frac{\sec^2 x}{tg\,x\sec^2 x + \sec^2 x}dx = \int \frac{1}{tg\,x+1}dx = \int \frac{1}{(t+1)(t^2+1)}dt =$

$t = tg\,x \to dt = \sec^2 x\,dx$

$= \int \frac{1-t}{2(t^2+1)}dt + \int \frac{1}{2(t+1)}dt = \frac{1}{2}\int \frac{1}{(t^2+1)}dt - \frac{1}{2}\int \frac{t}{(t^2+1)}dt + \frac{1}{2}\int \frac{1}{(t+1)}dt = \frac{1}{2}arc\,tg\,t - \frac{1}{4}\ln|t^2+1| + \frac{1}{2}\ln|t+1| + k =$

$= \frac{1}{2}x - \frac{1}{4}\ln|tg^2 x + 1| + \frac{1}{2}\ln|tg\,x+1| + k$

94. Página 290

$t = \cos^2 x - 2sen^2 x \to dt = -6\cos x\,sen\,x\,dx$

a) $\int \frac{sen x\cdot\cos x}{\cos^2 x - 2sen^2 x}dx = -\frac{1}{6}\int \frac{1}{t}dt = -\frac{1}{6}\ln|t| + k = -\frac{1}{6}\ln|\cos^2 x - 2sen^2 x| + k$

b) $\int \frac{1}{4-3\cos^2 x+5sen^2 x}dx = \int \frac{cosec^2 x}{(4-3\cos^3 x+5sen^2 x)cosec^2 x}dx = \int \frac{cosec^2 x}{4cosec^2 x - 3cotg^2 x + 5}dx = \int \frac{cosec^2 x}{9+cotg^2 x}dx =$

$t = cotg\,x \to dt = -cosec^2 x\,dx$

$= -\int \frac{1}{t^2+9}dt = \frac{1}{3}arc\,tg\left(\frac{t}{3}\right) + k = \frac{1}{3}arc\,tg\left(\frac{cotg\,x}{3}\right) + k$

c) $\int \frac{sen^3 x}{\sqrt[3]{\cos x}}\,dx = \int \frac{sen\,x(1-\cos^2 x)}{\sqrt[3]{\cos x}}dx = -\int \frac{1-t^2}{\sqrt[3]{t}}dt = -\int t^{-\frac{1}{3}}dt + \int t^{\frac{5}{3}}dt = -\frac{3}{2}\sqrt[3]{\cos^2 x} + \frac{3\sqrt[3]{\cos^8 x}}{8} + k$

$t = \cos x \to dt = -sen\,x\,dx$

$t = sen\,x \to dt = \cos x\,dx$

d) $\int \frac{\cos x}{2sen\,x\cos^2 x + sen^3 x}dx = \int \frac{\cos x}{sen^3 x + 2sen\,x(1-sen^2 x)}dx = \int \frac{\cos x}{2sen\,x - sen^3 x}dx = \int \frac{1}{2t-t^3}dt = \int\left(\frac{1}{2t} - \frac{t}{2(t^2-2)}\right)dt =$

$= \frac{1}{2}\ln|t| - \frac{1}{4}\ln|t^2-2| + k = \frac{1}{2}\ln|sen\,x| - \frac{1}{4}\ln|sen^2 x - 2| + k$

95. Página 290

a) $$\int \frac{1+\sqrt{x+1}}{1+\sqrt[3]{x+1}}dx = \int \frac{1+\left(\sqrt[6]{x+1}\right)^3}{1+\left(\sqrt[6]{x+1}\right)^2}dx = \int \frac{1+t^3}{1+t^2}6t^5dt = 6\int \frac{t^8+t^5}{1+t^2}dt = 6\int\left(t^6-t^4+t^3+t^2-t-1+\frac{t}{t^2+1}+\frac{1}{t^2+1}\right)dt =$$

$$t=\sqrt[6]{x+1} \to dt = \frac{1}{6\sqrt[6]{(x+1)^5}}dx \to dx = 6t^5dt$$

$$\frac{t^5}{5}+\frac{t^4}{4}+\frac{t^3}{3}-\frac{t^2}{2}-t+\frac{1}{2}\ln|t^2+1|+arctg\, t\Bigg)+k=$$

$$=6\left(\frac{\sqrt[6]{(x+1)}^7}{7}-\frac{\sqrt[6]{(x+1)}^5}{5}+\frac{\sqrt[3]{(x+1)}^2}{4}+\frac{\sqrt{(x+1)}}{3}-\frac{\sqrt[3]{(x+1)}}{2}-\sqrt[6]{(x+1)}+\frac{1}{2}\ln|\sqrt[3]{(x+1)}+1|+arctg\sqrt[6]{(x+1)}\right)+k$$

b) $$\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}dx = \int \frac{t^3}{1+t^2}\cdot 6t^5dt = \int \frac{6t^8}{1+t^2}dt = 6\int\left(t^6-t^4+t^2-1+\frac{1}{t^2+1}\right)dt = 6\left(\frac{t^7}{7}-\frac{t^5}{5}+\frac{t^3}{3}-t+arctg\, t\right)+k$$

$$t=\sqrt[6]{x} \to dt = \frac{1}{6\sqrt[6]{x^5}}dx \to dx = 6t^5dt$$

$$=6\left(\frac{\sqrt[6]{x^7}}{7}-\frac{\sqrt[6]{x^5}}{5}+\frac{\sqrt{x}}{3}-\sqrt[6]{x}+arctg\sqrt[6]{x}\right)+k$$

c) $$\int \frac{1+x+\sqrt{x+1}}{(x+1)\sqrt[3]{x+1}}dx = \int \frac{t^6+t^3}{t^6\cdot t^2}\cdot 6t^5dt = 6\int(t^3+1)dt = 6\left(\frac{t^4}{4}+t\right)+k = 6\left(\frac{\sqrt[6]{(1+x)^4}}{4}+\sqrt[6]{1+x}\right)+k=$$

$$t=\sqrt[6]{x+1} \to dt = \frac{1}{6\sqrt[6]{(x+1)^5}}dx \to dx = 6t^5dt$$

$$=6\left(\frac{\sqrt[3]{(1+x)^2}}{4}+\sqrt[6]{1+x}\right)+k$$

d) $$\int \frac{x+\sqrt{x}}{\sqrt{x}+\sqrt[4]{x}}dx = \int \frac{t^4+t^2}{t^2+t}\cdot 4t^3dt = 4\int \frac{t^6+t^4}{t+1}dt = 4\int\left(t^5-t^4+2t^3-2t^2+2t-2+\frac{2}{t+1}\right)dt =$$

$$t=\sqrt[4]{x} \to dt = \frac{1}{4\sqrt[4]{x^3}}dx \to dx = 4t^3dt$$

$$=4\left(\frac{t^6}{6}-\frac{t^5}{5}+\frac{2t^4}{4}-\frac{2t^3}{3}+\frac{2t^2}{2}-2t+2\ln|t+1|\right)+k = 4\left(\frac{\sqrt[4]{x^6}}{6}-\frac{\sqrt[4]{x^5}}{5}+\frac{x}{2}-\frac{2\sqrt[4]{x^3}}{3}+\sqrt{x}-2\sqrt[4]{x}+2\ln|\sqrt[4]{x}+1|\right)+k$$

e) $$\int \frac{1}{\sqrt{x}\left(1+\sqrt[4]{x}\right)}dx = \int \frac{1}{t^2(1+t)}\cdot 4t^3dt = 4\int \frac{t}{1+t}dt = 4\int\left(1+\frac{-1}{1+t}\right)dt = 4t-4\ln|t+1|+k = 4\sqrt[4]{x}-4\ln|\sqrt[4]{x}+1|+k$$

$$t=\sqrt[4]{x} \to dt = \frac{1}{4\sqrt[4]{x^3}}dx \to dx = 4t^3dt$$

f) $$\int \frac{1}{2+\sqrt{x+1}}dx = \int \frac{1}{2+t}\cdot 2t\,dt = 2\int\left(1+\frac{-2}{t+2}\right)dt = 2t-4\ln|t+2|+k = 2\sqrt{x+1}-4\ln|\sqrt{x+1}+2|+k$$

$$t=\sqrt{x+1} \to dt = \frac{1}{2\sqrt{x+1}}dx \to dx = 2t\,dt$$

96. Página 290

a) $$\int \frac{1}{\left(1-e^x\right)^2}dx = \int \frac{1}{(1-t)^2}\cdot\frac{1}{t}dt = \int\left(\frac{1}{t}-\frac{1}{t-1}+\frac{1}{(t-1)^2}\right)dt = \ln|t|-\ln|t-1|-\frac{1}{t-1}+k = x-\ln|e^x-1|-\frac{1}{e^x-1}+k$$

$$t=e^x \to dt = e^xdx \to dx =$$

$$t=e^x \to dt = e^xdx$$

b) $$\int \frac{1}{e^x+e^{-x}+1}dx = \int \frac{e^x}{e^{2x}+e^x+1}dx = \int \frac{1}{t^2+t+1}dt = \int \frac{1}{\left(t+\frac{1}{2}\right)^2+\frac{3}{4}}dt = \frac{4}{3}\int \frac{1}{\left(\frac{2}{\sqrt{3}}t+\frac{1}{\sqrt{3}}\right)^2+1}dt =$$

$$=\frac{4}{3}\frac{\sqrt{3}}{2}arctg\left(\frac{2}{\sqrt{3}}t+\frac{1}{\sqrt{3}}\right)+k = \frac{2}{\sqrt{3}}arctg\left(\frac{2}{\sqrt{3}}e^x+\frac{1}{\sqrt{3}}\right)+k$$

c) $\int \frac{e^{3x}}{e^{2x}-3e^{x}+2}dx = \int \frac{t^2}{t^2-3t+2}dt = \int\left(-\frac{1}{t-1}+\frac{4}{t-2}+1\right)dt = -\ln|t-1|+4\ln|t-2|+t+k =$

$t = e^x \rightarrow dt = e^x dx$

$= -\ln|e^x-1|+4\ln|e^x-2|+e^x+k$

d) $\int \frac{1+\sqrt{e^x}}{\left(1-\sqrt[4]{e^x}\right)^2}dx = \int \frac{1+t^2}{(1-t)^2}\cdot\frac{4}{t}dt = 4\int\frac{1+t^2}{t(1-t)^2}dt = 4\int\left(\frac{1}{t}+\frac{1}{1-t}+\frac{t+1}{(1-t)^2}\right)dt = 4\left(\ln|t|-\ln|1-t|+\int\frac{t-1+2}{(1-t)^2}dt\right)+k$

$t=\sqrt[4]{e^x} \rightarrow dt = \frac{\sqrt[4]{e^x}}{4}dx \rightarrow dx = \frac{4}{t}dt$

$= 4\left(\ln|t|-\ln|1-t|+\int\frac{-1}{1-t}dt+\int\frac{2}{(1-t)^2}dt\right)+k = 4\left(\ln|t|-\ln|1-t|+\ln|1-t|+\frac{2}{1-t}\right)+k =$

$= 4\left(\ln|\sqrt[4]{e^x}|+\frac{2}{1-\sqrt[4]{e^x}}\right)+k$

97. Página 290

a) $\int 2sen\,x\,e^{-cos\,x}\,dx = \int 2e^t dt = 2e^t+k = 2e^{-cos\,x}+k$

$t=-cos\,x \rightarrow dt = sen\,x\,dx$

b) $\int (1-x^2)^{-\frac{3}{2}}dx = \int \frac{cos\,t}{(1-sen^2 t)^{\frac{3}{2}}}dt = \int\frac{cos\,t}{cos^3 t}dt = \int\frac{1}{cos^2 t}dt = tg\,t+k = tg(arc\,sen\,x)+k = \frac{x}{\sqrt{1-x^2}}+k$

$x = sen\,t \rightarrow dx = cos\,t\,dt$

c) $\int x^5\sqrt{1-x^2}\,dx = \int sen^5 t\,cos^2 t = \int sen\,t(1-cos^2 t)^2 cos^2 t\,dt = -\int(1-u^2)^2u^2du = -\int(u^2-2u^4+u^6)du =$

$t = sen\,x \rightarrow dt = cos\,x$

$u = cos\,t \rightarrow du = -sen\,t\,dt$

$= -\frac{\ }{3}+\frac{\ }{5}-\frac{\ }{7}+k = -\frac{cos^3 t}{3}+\frac{2cos^5 t}{5}-\frac{cos^7 t}{7}+k = \left(-\frac{1}{3}cos\,t(1-sen^2 t)+\frac{2}{5}cos\,t(1-sen^2 t)^2-\frac{1}{7}cos\,t(1-sen^2 t)^3\right)+k =$

$= \sqrt{1-x^2}\cdot\left(-\frac{1-x^2}{3}+\frac{2(1-x^2)^2}{5}-\frac{(1-x^2)^3}{7}\right)+k$

d) $\int\left(1-(2x+1)^2\right)^{-\frac{1}{2}}dx = \frac{1}{2}\int(1-t^2)^{-\frac{1}{2}}dt = \frac{1}{2}\int\frac{1}{\sqrt{1-t^2}}dt = \frac{1}{2}arc\,sen(t)+k = \frac{1}{2}arc\,sen(2x+1)+k$

$t = 2x+1 \rightarrow dt = 2dx$

98. Página 290

a) $\int\frac{3x^2-5x+7}{x^3-4x^2+4x}dx = \int\left(\frac{9}{2(x-2)^2}+\frac{5}{4(x-2)}+\frac{7}{4x}\right)dx = \frac{-9}{2(x-2)}+\frac{5}{4}\ln|x-2|+\frac{7}{4}\ln|x|+k$

b) $\int\frac{x}{\sqrt{1-9x^2}}dx = -\frac{1}{18}\int\frac{-18x}{\sqrt{1-9x^2}}dx = -\frac{\sqrt{1-9x^2}}{9}+k$

c) $\int\frac{3x^2-7x+4}{2x-3}dx = \int\left(\frac{3}{2}x-\frac{5}{4}+\frac{1}{4(2x-3)}\right)dx = \frac{3x^2}{4}-\frac{5x}{4}+\frac{1}{8}\ln|2x-3|+k$

d) $\int \frac{2}{9+4x^2}dx = \int \frac{2}{9+(2x)^2}dx = \frac{1}{3}arc\,tg\frac{2x}{3}+k$

e) $\int \frac{5}{\sqrt{1-9x^2}}dx = \frac{5}{3}\int \frac{3}{\sqrt{1-(3x)^2}}dx = \frac{5}{3}arc\,sen\,3x+k$

f) $\int \frac{x}{\sqrt{1+x^2}}dx = \sqrt{1+x^2}+k$

g) $\int \sqrt{1-x^2}dx = \int \sqrt{1-sen^2 t}\cos t\,dt = \int \cos^2 t\,dt = \cos t\,sen\,t + \int sen^2 t\,dt =$

$x = sen\,t \rightarrow dx = \cos t\,dt$

$u = \cos t \rightarrow du = -sen\,t\,dt$
$dv = \cos t \rightarrow v = sen\,t$

$= \cos t\,sen\,t + \int (1-\cos^2 t)dt = \cos t\,sen\,t + t - \int \cos^2 t\,dt$

$\int \cos^2 t\,dt = \cos t\,sen\,t + t - \int \cos^2 t\,dt \rightarrow \int \cos^2 t\,dt = \frac{1}{2}(\cos t\,sen\,t + t)+k$

$\int \sqrt{1-x^2}dx = \frac{1}{2}\left(\frac{x\sqrt{1-x^2}+arc\,sen\,x}{2}\right)+k$

h) $\int \frac{x}{\sqrt{1+x}}dx = \int \frac{t-1}{\sqrt{t}}dt = \int \sqrt{t}dt - \int \frac{1}{\sqrt{t}}dt = \frac{2}{3}\sqrt{t^3}-2\sqrt{t}+k = \sqrt{1+x}\cdot\left(\frac{2(x+1)}{3}-2\right)+k$

$t = x+1 \rightarrow dt = dx$

99. Página 290

a) $\int \frac{x^3+4x^2-10x+7}{x^3-7x-6}dx = \int \left(1+\frac{2}{x-3}-\frac{5}{x+1}+\frac{7}{x+2}\right)dx =$

$= x + 2\ln|x-3| - 5\ln|x+1| + 7\ln|x+2| + k$

b) $\int \frac{1}{x^2-7x+10}dx = \int \left(\frac{1}{3(x-5)}-\frac{1}{3(x-2)}\right)dx = \frac{1}{3}\ln|x-5| - \frac{1}{3}\ln|x-2|+k$

c) $\int \frac{2^{3x}}{2^x-4}dx = \frac{1}{\ln 2}\int \frac{t^2}{t-4}dt = \frac{1}{\ln 2}\int \left(t+4+\frac{16}{t-4}\right)dt = \frac{1}{\ln 2}\left(\frac{t^2}{2}+4t+16\ln|t-4|\right)+k =$

$t = 2^x \rightarrow dt = 2^x \ln 2 dx \rightarrow dx = \frac{dt}{t\ln 2}$

$= \frac{1}{\ln 2}\left(\frac{2^{2x}}{2}+4\cdot 2^x + 16\ln|2^x-4|\right)+k$

d) $\int \frac{1}{x\sqrt{1-x}}dx = \int \frac{-2}{1-t^2}dt = \int \frac{-1}{1-t}dt - \int \frac{1}{1+t}dt = \ln|1-t| - \ln|1+t| + k = \ln|1-\sqrt{1-x}| - \ln|1+\sqrt{1-x}|+k$

$t = \sqrt{1-x} \rightarrow dt = \frac{-1}{2\sqrt{1-x}}dx$

e) $\int \frac{x+3}{4x^2+8}dx = \frac{1}{8}\int \frac{2x}{x^2+2}dx + \frac{1}{4}\int \frac{3}{x^2+2}dx = \frac{1}{8}\ln|x^2+2| + \frac{3}{4\sqrt{2}}arc\,tg\frac{x}{\sqrt{2}}+k$

f) $\int \frac{sen\,2x+\cos x}{\cos x}dx = \int \frac{2sen\,x\cos x+\cos x}{\cos x}dx = \int (2sen\,x+1)dx = -2\cos x + x + k$

100. **Página 290**

a) $\int cosec\, x\, dx = \int \frac{1}{sen\, x} dx = \int \frac{-1}{1-t^2} dt = \frac{-1}{2}\ln|1+t| + \frac{1}{2}\ln|1-t| + k = -\frac{1}{2}\ln|1+\cos x| + \frac{1}{2}\ln|1-\cos x| + k$

$t = \cos x \to dt = -sen\, x\, dx$

b) $\int \sec x\, dx = \int \frac{1}{\cos x} dx = \int \frac{1}{1-t^2} dt = \frac{1}{2}\ln|1+t| - \frac{1}{2}\ln|1-t| + k = \frac{1}{2}\ln|1+sen\, x| - \frac{1}{2}\ln|1-sen\, x| + k$

$t = sen\, x \to dt = \cos x\, dx$

c) $\int e^{x^2-5x}(2x-5)\, dx = e^{x^2-5x} + k$

d) $\int \frac{sen\, x}{\sqrt{1+\cos x}} dx = -2\sqrt{1+\cos x} + k$

e) $\int \frac{1}{x\sqrt{1-(\ln x)^2}} dx = arc\, sen(\ln x) + k$

f) $\int \frac{1+e^{3x}}{e^{2x}} dx = \int \frac{1}{e^{2x}} dx + \int e^x dx = \frac{-e^{-2x}}{2} + e^x + k$

g) $\int \sqrt{e^x - 1}\, dx = \int (t+1)\sqrt{t}\, dt = \frac{2}{5}t^{\frac{5}{2}} + \frac{2}{3}t^{\frac{3}{2}} + k = \sqrt{e^x-1}\left(\frac{2}{5}(e^x-1)^2 + \frac{2}{3}(e^x-1)\right) + k$

$t = e^x - 1 \to dt = e^x dx$

h) $\int \frac{sen\, 2x}{1+\cos^2 x} dx = \int \frac{2\, sen\, x \cos x}{1+\cos^2 x} dx = -\int \frac{-2\, sen\, x \cos x}{1+\cos^2 x} dx = -2\ln|1+\cos^2 x| + k$

101. **Página 290**

a) $\int (x-2)e^{3x} dx = \frac{1}{3}(x-2)e^{3x} - \int \frac{e^{3x}}{3} dx = \frac{1}{3}(x-2)e^{3x} - \frac{e^{3x}}{9} + k$

$u = x - 2 \to du = dx$

$dv = e^{3x} dx \to v = \frac{e^{3x}}{3}$

b) $\int \frac{7}{x\sqrt[3]{\ln x}} dx = \frac{21}{2}\sqrt[3]{(\ln x)^2} + k$

c) $\int \frac{(\ln x)^2 + x}{x} dx = \int \frac{(\ln x)^2}{x} dx + \int dx = \frac{(\ln x)^3}{3} + x + k$

d) $\int (\ln x)^2 dx = (x\ln x - x)\ln x - \int (\ln x - 1) dx = (x\ln x - x)\ln x - x\ln x + 2x + k =$

$u = \ln x \to du = \frac{dx}{x}$

$dv = \ln x\, dx \to v = x\ln x - x$

$= x(\ln x)^2 - 2x\ln x + 2x + k$

e) $\int \frac{5e^{2x} - e^x}{e^{2x} - 1} dx = \int \frac{5t-1}{t^2-1} dt = \int \frac{2}{t-1} dt + \int \frac{3}{t+1} dt = 2\ln|t-1| + 3\ln|t+1| + k =$

$t = e^x \to dt = e^x dx$

$= 2\ln|e^x - 1| + 3\ln|e^x + 1| + k$

f) $\int sen^2 x\, dx = \int \frac{1-\cos 2x}{2} dx = \frac{x}{2} - \frac{sen\, 2x}{4} + k$

g) $\int \cos^2 x\, dx = \int \frac{1+\cos 2x}{2} dx = \frac{x}{2} + \frac{sen\, 2x}{4} + k = \frac{x + sen\, x \cos x}{2} + k$

h) $\int sen\, x \cos x\, dx = \frac{sen^2 x}{2} + k$

102. Página 290

a) $\int 2^x \cos x\,dx = 2^x \operatorname{sen} x - \ln 2 \int 2^x \operatorname{sen} x\,dx = 2^x \operatorname{sen} x + \ln 2 \cdot 2^x \cos x - (\ln 2)^2 \int 2^x \cos x\,dx$

$u = 2^x \rightarrow du = 2^x \ln 2 dx$
$dv = \cos x\,dx \rightarrow v = \operatorname{sen} x$

$u = 2^x \rightarrow du = 2^x \ln 2 dx$
$dv = \operatorname{sen} x\,dx \rightarrow v = -\cos x$

$\int 2^x \cos x\,dx = \dfrac{2^x \operatorname{sen} x + \ln 2 \cdot 2^x \cos x}{1 + (\ln 2)^2}$

b) $\int \dfrac{\ln x + 3}{x(\ln x - 1)}dx = \int \dfrac{t+3}{t-1}dt = t + 4\ln|t-1| + k = \ln|x| + 4\ln|\ln x - 1| + k$

$t = \ln x \rightarrow dt = \dfrac{dx}{x}$

c) $\int \cos(\ln x)dx = \int e^t \cos t\,dt = e^t \operatorname{sen} t - \int e^t \operatorname{sen} t\,dt = e^t \operatorname{sen} t + e^t \cos t - \int e^t \cos t\,dt$

$t = \ln x \rightarrow dt = \dfrac{dx}{x}$

$u = e^t \rightarrow du = e^t\,dt$
$dv = \cos t\,dt \rightarrow v = \operatorname{sen} t$

$u = e^t \rightarrow du = e^t\,dt$
$dv = \operatorname{sen} t\,dt \rightarrow v = -\cos t$

$\int \cos(\ln x)dx = \dfrac{e^t \operatorname{sen} t + e^t \cos t}{2} + k = \dfrac{x \operatorname{sen}(\ln x) + x\cos(\ln x)}{2} + k$

d) $\int \operatorname{sen}\sqrt{x}\,dx = 2\int t \operatorname{sen} t\,dt = 2\left(-t\cos t + \int \cos t\,dt\right) = -2t\cos t + 2\operatorname{sen} t + k = -2\sqrt{x}\cos\sqrt{x} + 2\operatorname{sen}\sqrt{x} + k$

$t = \sqrt{x} \rightarrow dt = \dfrac{dx}{2\sqrt{x}}$

$u = t \rightarrow du = dt$
$dv = \operatorname{sen} t\,dt \rightarrow v = -\cos t$

e) $\int \operatorname{sen}^2 x \cos x\,dx = \dfrac{\operatorname{sen}^3 x}{3} + k$

f) $\int \operatorname{sen}^3 x \cos x\,dx = \dfrac{\operatorname{sen}^4 x}{4} + k$

g) $\int \operatorname{sen}^3 x\,dx = \int (1 - \cos^2 x)\operatorname{sen} x\,dx = \int \operatorname{sen} x\,dx - \int \cos^2 x \operatorname{sen} x\,dx = -\cos x + \dfrac{\cos^3 x}{3} + k$

h) $\int \cos^3 x\,dx = \int (1 - \operatorname{sen}^2 x)\cos x\,dx = \int \cos x\,dx - \int \operatorname{sen}^2 x \cos x\,dx = \operatorname{sen} x - \dfrac{\operatorname{sen}^3 x}{3} + k$

103. Página 291

a) $\int x \operatorname{sen}(\ln x)dx = \dfrac{x^2}{2}\operatorname{sen}(\ln x) - \dfrac{1}{2}\int x\cos(\ln x)dx = \dfrac{x^2}{2}\operatorname{sen}(\ln x) - \dfrac{x^2}{4}\cos(\ln x) - \dfrac{1}{4}\int x \operatorname{sen}(\ln x)dx$

$u = \operatorname{sen}(\ln x) \rightarrow du = \dfrac{\cos(\ln x)}{x}dx$
$dv = x\,dx \rightarrow v = \dfrac{x^2}{2}$

$u = \cos(\ln x) \rightarrow du = \dfrac{-\operatorname{sen}(\ln x)}{x}dx$
$dv = x\,dx \rightarrow v = \dfrac{x^2}{2}$

$\dfrac{5}{4}\int x \operatorname{sen} x(\ln x)dx = \dfrac{x^2}{2}\operatorname{sen}(\ln x) - \dfrac{x^2}{4}\cos(\ln x) + k \rightarrow \int x \operatorname{sen} x(\ln x)dx = \dfrac{2x^2 \operatorname{sen}(\ln x) - x^2\cos(\ln x)}{5} + k$

b) $\int \operatorname{tg} x \sec^2 x\,dx = \int \dfrac{\operatorname{sen} x}{\cos^3 x}dx = \dfrac{1}{2\cos^2 x} + k$

c) $\int \dfrac{\cos^3 x}{\operatorname{sen} x}dx = \int \dfrac{1-t^2}{t}dt = \ln|t| - \dfrac{t^2}{2} + k = \ln|\operatorname{sen} x| - \dfrac{\operatorname{sen}^2 x}{2} + k$

$t = \operatorname{sen} x \rightarrow dt = \cos x\,dx$

d) $\int (\cos^2 x - \operatorname{sen} x \cos^2 x)\,dx = \int \cos^2 x\,dx - \int \operatorname{sen} x \cos^2 x\,dx = \int \frac{1+\cos 2x}{2}dx - \int \operatorname{sen} x \cos^2 x\,dx =$

$= \frac{x}{2} + \frac{\operatorname{sen} 2x}{4} - \frac{\cos^3 x}{3} + k$

e) $\int \frac{\cos x - \operatorname{sen} x}{2}dx = \int \frac{\cos x}{2}dx - \int \frac{\operatorname{sen} x}{2}dx = \frac{\operatorname{sen} x + \cos x}{2} + k$

f) $\int \frac{\cos^2 x \operatorname{sen} x + \cos x \operatorname{sen}^2 x}{\operatorname{sen} x}dx = \int (\cos^2 x + \cos x \operatorname{sen} x)\,dx = \int \cos^2 x\,dx + \int \cos x \operatorname{sen} x\,dx =$

$= \int \frac{1+\cos 2x}{2}dx + \int \cos x \operatorname{sen} x\,dx = \frac{x}{2} + \frac{\operatorname{sen} 2x}{4} + \frac{\operatorname{sen}^2 x}{2} + k$

104. Página 291

a) $\int x^3\sqrt{2x+1}\,dx = \frac{1}{2}\int \left(\frac{t-1}{2}\right)^3 \cdot \sqrt{t}\,dt = \frac{1}{16}\int (t-1)^3\sqrt{t}\,dt = \frac{1}{16}\int (t^{7/2} - 3t^{5/2} + 3t^{3/2} - t^{1/2})\,dt =$

$t = 2x+1 \rightarrow dt = 2dx$

$= \frac{1}{16}\left(\frac{2}{9}t^{9/2} - \frac{6}{7}t^{7/2} + \frac{6}{5}t^{5/2} - \frac{2}{3}t^{3/2}\right) + k = \frac{1}{16}\left(\frac{2}{9}(2x+1)^{9/2} - \frac{6}{7}(2x+1)^{7/2} + \frac{6}{5}(2x+1)^{5/2} - \frac{2}{3}(2x+1)^{3/2}\right) + k$

b) $\int \frac{-2x+6}{x^3 - 2x^2 - x + 2}dx = \int \left(-\frac{2}{x-1} + \frac{4}{3(x+1)} + \frac{2}{3(x-2)}\right) = -2\ln|x-1| + \frac{4}{3}\ln|x+1| + \frac{2}{3}\ln|x-2| + k$

c) $\int \frac{\operatorname{sen} 2x}{\sqrt{1+\cos 2x}}dx = -\frac{1}{2}\int \frac{1}{\sqrt{t}}dt = -\sqrt{t} + k = -\sqrt{1+\cos 2x} + k$

$t = 1 + \cos 2x \rightarrow dt = -2\operatorname{sen} 2x\,dx$

d) $\int \frac{1}{x^4\sqrt{x^2-1}}dx = \int \frac{\operatorname{tg} t \sec t}{\sec^4 t \cdot \sqrt{\sec^2 t - 1}}dt = \int \frac{\operatorname{tg} t \sec t}{\sec^4 t \cdot \operatorname{tg} t}dt = \int \frac{1}{\sec^3 t}dt = \int \cos^3 t\,dt = \int (1 - \operatorname{sen}^2 x)\cos x\,dx =$

$x = \sec t \rightarrow dx = \operatorname{tg} t \sec t\,dt$

$= \int \cos x\,dx - \int \operatorname{sen}^2 x \cos x\,dx = \operatorname{sen} x - \frac{\operatorname{sen}^3 x}{3} + k$

105. Página 291

a) $\int 4\operatorname{sen} 3x \operatorname{sen} 2x\,dx = 4\int \operatorname{sen} 3x \operatorname{sen} 2x\,dx = 4\int \frac{1}{2}(\cos(-x) - \cos(5x))\,dx = -2\operatorname{sen}(-x) - \frac{2}{5}\operatorname{sen}(5x) + k$

b) $\int \frac{(2+x)^2}{x(4+x^2)}dx = \int \left(\frac{4}{x^2+4} + \frac{1}{x}\right)dx = \int \frac{1}{\left(\frac{x}{2}\right)^2 + 1}dx + \int \frac{1}{x}dx = 2\operatorname{arctg}\left(\frac{x}{2}\right) + \ln|x| + k$

c) $\int -3\operatorname{sen} 2x \cos x\,dx = -3\int \frac{1}{2}(\operatorname{sen} x + \operatorname{sen} 3x)\,dx = \frac{3}{2}\cos x + \frac{1}{2}\cos 3x + k$

d) $\int \frac{3x^2+5}{2x^2+4}dx = \int \left(\frac{3}{2} - \frac{1}{2(x^2+2)}\right)dx = \int \frac{3}{2}dx - \frac{1}{4}\int \frac{1}{\left(\frac{x}{\sqrt{2}}\right)^2 + 1}dx = \frac{3}{2}x - \frac{\sqrt{2}}{4}\operatorname{arctg}\left(\frac{x}{\sqrt{2}}\right) + k$

106. Página 291

a) $\int \frac{1}{\sqrt{x+2}+\sqrt{x-2}}dx = \int \frac{1}{4}\left(\sqrt{x+2}-\sqrt{x-2}\right)dx = \frac{1}{6}\sqrt{(x+2)^3}-\frac{1}{6}\sqrt{(x-2)^3}+k$

b) $\int \frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1-x^4}}dx = \int \frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{(1+x^2)(1-x^2)}}dx = \int \frac{1}{\sqrt{1+x^2}}dx + \int \frac{1}{\sqrt{1-x^2}}dx =$

$= arc\,senh(x) + arc\,sen(x) + k$

c) $\int \frac{x^4+5\sqrt[3]{x}-3x\sqrt{x}-2}{4x}dx = \frac{1}{4}\int \left(x^3+5x^{-\frac{2}{3}}-3\sqrt{x}-2x^{-1}\right)dx = \frac{1}{4}\left(\frac{1}{4}x^4+15\sqrt[3]{x}-2\sqrt{x^3}-2\ln|x|\right)+k$

d) $\int \left(x-x^{-3}\right)\sqrt{x\sqrt{x\sqrt{x}}}\,dx = \int \left(x-x^{-3}\right)\cdot x^{\frac{1}{2}}\cdot x^{\frac{1}{4}}\cdot x^{\frac{1}{8}}\,dx = \int \left(x-x^{-3}\right)\cdot x^{\frac{7}{8}}\,dx = \int \left(x^{\frac{15}{8}}-x^{-\frac{17}{8}}\right)dx = \frac{8}{23}x^{\frac{23}{8}}+\frac{8}{9}x^{-\frac{9}{8}}+k$

$= \frac{8}{23}\sqrt[8]{x^{23}}+\frac{8}{9\sqrt[8]{x^9}}+k$

107. Página 291

a) $\int f(x)dx = \int \frac{3}{\sqrt[3]{x^2}}dx = 3\int x^{-\frac{2}{3}}dx = 9\sqrt[3]{x}+k$

b) $\int g(x)dx = \int \frac{x+5}{x^2+x-2}dx = \int \left(\frac{2}{x-1}-\frac{1}{x+2}\right)dx = 2\ln|x-1|-\ln|x+2|+k$

c) $\int h(x)dx = \int \frac{\cos x}{1+sen^2 x}dx = \int \frac{1}{1+t^2}dt = arc\,tg\,t + k = arc\,tg(sen\,x)+k$

$t = sen\,x \rightarrow dt = \cos x\,dx$

d) $\int i(x)dx = \int \frac{1}{4+x^2}dx = \frac{1}{4}\int \frac{1}{1+\left(\frac{x}{2}\right)^2}dx = \frac{1}{2}arc\,tg\left(\frac{x}{2}\right)+k$

108. Página 291

a) $\int \frac{sen\,5x}{\cos^2 5x}dx = -\frac{1}{5}\int \frac{1}{t^2}dt = \frac{1}{5t}+k = \frac{1}{5\cos 5x}+k$

$t = \cos 5x \rightarrow dt = -5sen\,5x\,dx$

b) $\int \frac{3x+2}{x^2+8x+7}dx = \int \left(\frac{19}{6(x+7)}-\frac{1}{6(x+1)}\right)dx = \frac{19}{6}\ln|x+7|-\frac{1}{6}|x+1|+k$

c) $\int x^2\cdot\sqrt{3x^3+7}\,dx = \frac{1}{9}\int \sqrt{t}+k = \frac{1}{9}\cdot\frac{2}{3}\cdot t^{\frac{3}{2}}+k = \frac{2}{27}\sqrt{(3x^3+7)^3}+k$

$t = 3x^3+7 \rightarrow dt = 9x^2\,dx$

d) $\int \left(\frac{\sqrt{x}}{3x}-\frac{5x}{\sqrt[3]{x}}\right)dx = \int \left(\frac{1}{3}x^{-\frac{1}{2}}-5x^{\frac{2}{3}}\right)dx = \frac{2}{3}x^{\frac{1}{2}}-3x^{\frac{5}{3}}+k = \frac{2\sqrt{x}}{3}-5\sqrt[3]{x^5}+k$

109. Página 291

a) $\int \frac{3e^x + e^{3x}}{e^x} dx = \int \frac{3e^x}{e^x} dx + \int \frac{e^{3x}}{e^x} dx = 3\int dx + \int e^{2x} dx = 3x + \frac{1}{2}e^{2x} + k$

b) $\int \frac{e^x}{1-e^{2x}} dx = \int \frac{1}{1-t^2} dt = \int \left(\frac{1}{2(t+1)} - \frac{1}{2(t-1)} \right) dt = \frac{1}{2}\ln|t+1| - \frac{1}{2}\ln|t-1| + k = \frac{1}{2}\ln|e^x+1| - \frac{1}{2}\ln|e^x-1| + k$

$t = e^x \to dt = e^x dx$

c) $\int \frac{x-1}{\sqrt{2x}-\sqrt{x+1}} dx = \int \frac{(x-1)\left(\sqrt{2x}+\sqrt{x+1}\right)}{x-1} dx = \int \left(\sqrt{2x}+\sqrt{x+1}\right) dx = \frac{2\sqrt{2}}{3}\sqrt{x^3} + \frac{2}{3}\sqrt{(x+1)^3} + k$

110. Página 291

Si $a = 0$, entonces $\int \frac{a}{\sqrt{a^2-x^2}} dx = k$.

Si $a \neq 0$, entonces $\int \frac{a}{\sqrt{a^2-x^2}} dx = \int \frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}} dx = a \cdot arc\,sen\left(\frac{x}{a}\right) + k$.

111. Página 291

a) $\int \frac{1+(\ln x)^3}{x(\ln^4 x + \ln^2 x)} dx = \int \frac{1+t^3}{t^4+t^2} dt = \int \left(\frac{t-1}{t^2+1} + \frac{1}{t^2} \right) dt = \int \left(\frac{t}{t^2+1} - \frac{1}{t^2+1} + \frac{1}{t^2} \right) dt = \frac{1}{2}\ln|t^2+1| - arc\,tg\,t - \frac{1}{t} + k =$

$t = \ln x \to dt = \frac{1}{x} dx$

$= \frac{1}{2}\ln|(\ln x)^2+1| - arc\,tg(\ln x) - \frac{1}{\ln x} + k$

b) $\int e^x \left[e^x \cdot sen\left(e^x\right)\right] dx = \int t\,sen\,t\,dt = -t\cos t - \int \cos t\,dt = -t\cos t - sen\,t + k = -e^x \cos e^x - sen\,e^x + k$

$t = e^x \to dt = e^x dx$

112. Página 291

Si $a \neq 2$:

$\int \frac{3dx}{x^2-(a+2)x+2a} = \int \frac{3dx}{(x-a)(x-2)} = -\frac{3}{2-a}\int \frac{dx}{x-a} + \frac{3}{2-a}\int \frac{dx}{x-2} = -\frac{3}{2-a}\ln|x-a| + \frac{3}{2-a}\ln|x-2| + k$

Si $a = 2$:

$\int \frac{3}{(x-2)^2} dx = \int 3(x-2)^{-2} dx = \frac{-3}{x-2} + k$

113. Página 291

$\int (1-\cos^2 x) \cdot sen\,2x \cdot e^{\cos^2 x} dx = \int (1-\cos^2 x) \cdot 2\,sen\,x\cos x \cdot e^{\cos^2 x} dx = -\int (1-t)e^t dt = -\int e^t dt + \int te^t dt =$

$t = \cos^2 x \to dt = -2\cos x\,sen\,x\,dx$

$u = t \to du = dt$
$dv = e^t dt \to v = e^t$

$= -e^t + \left[te^t - \int e^t dt\right] = -e^t + te^t - e^t + k = e^{\cos^2 x}(\cos^2 x - 2) + k$

114. Página 291

$$F(x)=\int f(x)dx=\int x\cdot e^{2x}dx=\frac{1}{2}xe^{2x}-\frac{1}{2}\int e^{2x}dx=\frac{1}{2}xe^{2x}-\frac{1}{4}e^{2x}+k$$

$$F(0)=2\rightarrow -\frac{1}{4}+k=2\rightarrow k=\frac{9}{4}$$

La función que cumple estas condiciones es: $F(x)=\frac{1}{2}xe^{2x}-\frac{1}{4}e^{2x}+\frac{9}{4}$

115. Página 291

a) $f(x)=\int f'(x)dx=\begin{cases} f_1(x)=\int(3x^2-2)dx & \text{si} \quad x\le 1 \\ f_2(x)=\int(1+\ln x)dx & \text{si} \quad x>1 \end{cases}$

$$f_1(x)=\int(3x^2-2)dx=x^3-2x+k\rightarrow f_1(-1)=2\rightarrow$$

$$-1+2+k=2\rightarrow k=1\rightarrow f_1(x)=x^3-2x+1$$

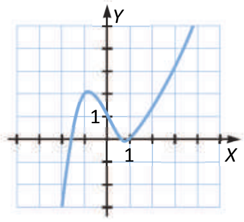
$$f_2(x)=\int(1+\ln x)dx=x+\int \ln x dx=$$

$$=x+x\ln x-\int dx=x+x\ln x-x=x\ln x+k$$

F debe ser continua en *x* = 1, entonces: $f_1(1)=f_2(1)\rightarrow 1-2+1=1\cdot\ln 1+k\rightarrow k=0\rightarrow f_2(x)=x\ln x$

$$f(x)=\int f'(x)dx=\begin{cases} f_1(x)=x^3-2x+1 & \text{si} \quad x\le 1 \\ f_2(x)=x\ln x & \text{si} \quad x>1 \end{cases}$$

b)



116. Página 291

$$f(x)=\int f'(x)dx=\begin{cases} f_1(x)=\int \frac{-1}{\sqrt{-x}}dx & \text{si} \quad x\le -1 \\ f_2(x)=\int(5x^4-6x^2)dx & \text{si} \quad x>-1 \end{cases}$$

$$f_1(x)=\int \frac{-1}{\sqrt{-x}}dx=2\sqrt{-x}+k$$

$$f_2(x)=\int(5x^4-6x^2)dx=x^5-2x^3+k\rightarrow f_2(2)=15\rightarrow 32-16+k=15\rightarrow k=-1\rightarrow f_2(x)=x^5-2x^3-1$$

F debe ser continua en x = –1, entonces: $f_1(-1)=f_2(-1)\rightarrow 2+k=0\rightarrow k=-2\rightarrow f_1(x)=2\sqrt{-x}-2$

$$f(x)=\begin{cases} f_1(x)=2\sqrt{-x}-2 & \text{si} \quad x\le -1 \\ f_2(x)=x^5-2x^3-1 & \text{si} \quad x>-1 \end{cases}$$

117. Página 291

$$\int \frac{-3}{(3x+a)^2}dx=-\int 3(3x+a)^{-2}dx=\frac{1}{3x+a}+k$$

a) Para que *y* = 4 sea asíntota, *k* debe valer 4.

Para que el eje de abscisas (*y* = 0) sea asíntota, *k* debe valer 0.

b) Para que *x* = 1 sea asíntota, *a* debe valer –3.

Para que el eje de ordenadas (*x* = 0) sea asíntota, *a* debe valer 0.

MATEMÁTICAS EN TU VIDA

1. Página 292

El beneficio viene dado por: $R(x) = 2300 - (x - 50)^2$

Vendiendo 30 pares: $R(30) = 2300 - (30 - 50)^2 = 1900$

Vendiendo 25 pares: $R(25) = 2300 - (25 - 50)^2 = 1675$

2. Página 292

Con la venta de 50 pares de zapatillas se obtiene el beneficio máximo, por lo que si los precios no varían, los beneficios empezarían a disminuir.

Si se venden menos de 50 pares, la empresa obtiene beneficios, pero no llegan al beneficio máximo.

3. Página 292

Veamos para qué valores de *x* la función de beneficio es positiva. Para ello, buscaremos los puntos en los que dicha función se anula:

$$R(x) = 0 \rightarrow 2300 - (x - 50)^2 = 0 \Leftrightarrow \begin{cases} x_1 = -10(\sqrt{23} - 5) \simeq 2{,}04 \\ x_2 = 10(5 + \sqrt{23}) \simeq 97{,}96 \end{cases}$$

La función de beneficio se anula en $x = 2{,}04$ y en $x = 97{,}96$. Comprobemos que en valores intermedios la función es positiva, tomando, por ejemplo, $x = 10$.

$R(10) = 700 > 0$

Tenemos, por tanto, que la función de beneficio toma valores positivos en el intervalo $(2{,}04;\ 97{,}96)$, pero como estamos trabajando con pares de zapatos, los valores deben ser enteros, por lo que diremos que obtenemos beneficio en el intervalo $[3,\ 97]$.

4. Página 292

Como ya hemos hallado el intervalo en el que se obtiene beneficio, el mínimo beneficio se obtendrá en alguno de los extremos del intervalo. Veamos en cuál:

$R(3) = 91 = R(97)$

En ambos extremos se obtiene el mismo beneficio, que es de 91 €.